$$a_1^n u^n{}_{,xx} + a_6^n \psi^n{}_{,xx} - a_7^n w^n{}_{,xxx} = 0, \quad a_6^n u^n{}_{,xx} + a_2^n \psi^n{}_{,xx} - a_3^n w^n{}_{,xxx} - a_5^n \psi^n = 0,$$
$$a_7^n u^n{}_{,xxx} + a_3^n \psi^n{}_{,xxx} - a_4^n w^n{}_{,xxxx} - \kappa^n w^n = 0 \ \ (n = I, III). \tag{3}$$

Для участка *II*, не связанного с упругим основанием и нагруженного поперечной нагрузкой, система имеет вид:

$$a_1^n u^n{}_{,xx} + a_6^n \psi^n{}_{,xx} - a_7^n w^n{}_{,xxx} = 0, \quad a_6^n u^n{}_{,xx} + a_2^n \psi^n{}_{,xx} - a_3^n w^n{}_{,xxx} - a_5^n \psi^n = 0,$$
$$a_7^n u^n{}_{,xxx} + a_3^n \psi^n{}_{,xxx} - a_4^n w^n{}_{,xxxx} = -q^n, \ (n = II). \tag{4}$$

Здесь a_1^n, …, a_7^n – параметры, характеризующие геометрические и упругие свойства слоев на *n*-м участке,

$$a_1^n = A_x^{(1)} h_1^n + A_x^{(2)} h_2^n + 2A_x^{(3)} c^n \ ; \ a_2^n = c^{n2}\left[A_x^{(1)} h_1^n + A_x^{(2)} h_2^n + \tfrac{2}{3} A_x^{(3)} c^n\right]; \ \ldots \tag{5}$$

Соотношения для сдвига в заполнителе $\psi^n(x)$, прогиба $w^n(x)$ и продольного перемещения срединной плоскости заполнителя $u^n(x)$ получим, решив соответствующие системы уравнений равновесия (3), (4). Аналитический вид решений зависит от типа упругого основания.

Для первого и третьего участков ($n = I, III$) на упругом основании средней жесткости выражение для прогиба имеет вид

$$w^n(x) = C_1^n \operatorname{sh}(\lambda_1^n x) + C_2^n \operatorname{ch}(\lambda_1^n x) + C_3^n \operatorname{sh}(\lambda_3^n x) + C_4^n \operatorname{ch}(\lambda_3^n x) + C_5^n \operatorname{sh}(\lambda_5^n x) + C_6^n \operatorname{ch}(\lambda_5^n x) + w_p^n. \tag{6}$$

Для второго участка ($n = II$), не связанного с упругим основанием,

$$w^n(x) = C_1^n \left(\alpha_{17}^n b_{11}^n x + \alpha_{18}^n x^3 / 6\right) + C_2^n b_{14}^n \operatorname{ch}(\beta_3^n x) + C_3^n b_{14}^n \operatorname{sh}(\beta_3^n x) + C_4^n x^2/2 + C_5^n x + C_6^n + g_2^n(x). \tag{7}$$

В выражениях (6), (7) C_1^n, ..., C_6^n – константы интегрирования, $w_p^n(x) = \frac{q}{\kappa}(a_0 x + b_0)$ – частное решение, соответствующее линейной нагрузке (1), коэффициенты $b_i^n, \alpha_i^n, \beta_i^n$ и функция $g_2^n(x)$ выражаются через параметры a_i^n (5).

Для определения констант интегрирования необходимо учесть граничные условия и условия сопряжения участков друг с другом. Граничные условия на торцах и условия сопряжения на границах участков позволяют составить систему 24-линейных алгебраических уравнений для определения констант интегрирования.

Объединением решений для отдельных участков получаются перемещения для всей пластины. Выражение для прогиба имеет вид

$$w(x) = w^I(x) + \left[w^{II}(x) - w^I(x)\right]\mathrm{H}_0(x - x_1) + \left[w^{III}(x) - w^{II}(x)\right]\mathrm{H}_0(x - x_2). \tag{8}$$

В выражении (8) H_0 – функция Хевисайда.

Был составлен комплекс программ в среде MathCad, с помощью которого получены числовые результаты для нескольких типов трехслойных пакетов.

Работа выполнена при финансовой поддержке БРФФИ (проект T20P-047).

УДК 539.8

MECHANICAL RESPONSE OF A STRUCTURAL ELEMENT CONTAINING A HEALING AGENT: FINITE ELEMENT MODELING

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Various types of composites find numerous applications in all fields of industry: aeronautical, automotive, building, etc. Generally, composite materials are a viable technical solution because they are lightweight, and, at the same time, they possess high strength. Also, they are resistant to corrosion, chemical attacks, and high temperature.

This study focuses on the numerical modeling of the structural element made of a cement-based composite containing a healing agent. Presumably, nucleation and propagation of cracks trigger the self-healing process. The propagating crack could, for example, intersect the encapsulated components of the healing agent, provoking thus their interaction and subsequent hardening of the healing substance. As a result, the structural element might regain partially its initial stiffness and load-carrying capacity.

A damage mechanics-based approach is chosen to simulate the strain-softening behavior of the cement-based composite, specifically, elasticity coupled with damage,

$$\sigma_{ij} = \frac{\nu}{(1+\nu)(1-2\nu)} E(1-D)\varepsilon_{kk}\delta_{ij} + \frac{\nu}{(1+\nu)} E(1-D)\varepsilon_{ij}, . \tag{1}$$

where σ_{ij} – are the components of the stress tensor, ε_{ij} – the components of the strain tensor, ν – is the Poisson’s ratio, E – the Yong’s modulus, and D – the damage variable. The damage variable that affects the material properties of the cement-based composite is defined as follows,

$$D = f\left(C_i, \varepsilon_{eqv}\right). \tag{2}$$

In equation (2), C_i are model constants that should be identified based on experimental data, and the equivalent strain is calculated based on the positive principal strains [1]

$$\varepsilon_{eqv} = \sqrt{\sum_{i=1}^{3} \left\langle \varepsilon_i \right\rangle}, \tag{3}$$

as the McAuley brackets are classically defined,

$$\left\langle \varepsilon_i \right\rangle = \frac{1}{2}\left(\varepsilon_i + \left|\varepsilon_i\right|\right). \tag{4}$$

The developed customized numerical procedure integrates the constitutive relationship that models the strain-softening response of the cement-based composite into a general-purpose finite element code. For each increment of the applied load (within the force-controlled or displacement controlled simulation), the stress and the strain distribution within the structural element are obtained by nonlinear finite element analysis. The damage variable is calculated based on the obtained stress and strain distributions. For the subsequent analysis, material properties of the finite elements affected by damage are modified, taking into account the accumulated mechanical damage modeling thus the strain-softening behavior of the cement-based composite (Figure 1).

Finite elements, in which critical damage is detected, are deactivated. In this way, zones of zero rigidity form in the structural element, modeling thus the crack initiation and propagation.

Under the assumption that the self-healing effect is triggered if a specific width of the initiated crack is reached, the healing of the newly-formed cracks is modeled through the homogenization of the system 'cement-based composite – hardened healing agent'.

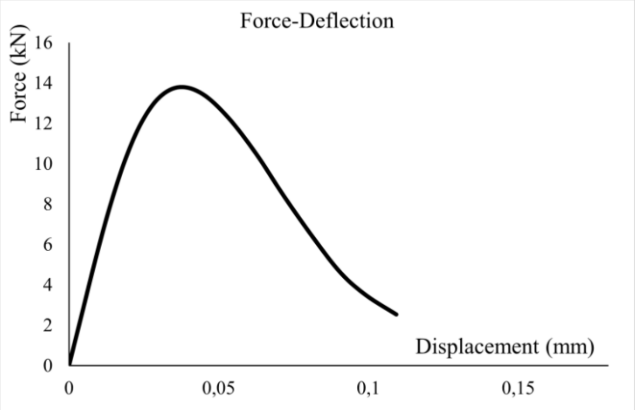

Figure 1 – ‘Force – mid-span deflection’ relationship obtained by finite element analysis: simulation of a displacement-controlled quasi-static four-point bending test

The next steps in this ongoing study are summarized as follows:

– definition of a constitutive relationship capable to account for the potential regain in the stiffness of the cement-based composites resulting from the possible crack reclosures in repetitive loading paths (hysteresis cycles);

– calibration of the strain-softening constitutive relation, based on experimental data obtained in characterization tests, such as compression tests or tension (by flexure) tests performed on small-scale specimens made of cement-based composite;

– validation of the proposed model (integrated into a general-purpose finite element code) through comparison with benchmark examples (experimental studies) found in the literature;

– definition of dependency (for practical use) between the crack width, the accumulated damage, evaluated trough the damage variable, and the effects of various types of healing agents.

References

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