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Э. ЧИСТИНА, М. ХАМАДИШ, Н. КИЗИЛОВА

СТАБИЛИЗАЦИЯ ТЕЧЕНИЙ ПО МНОГОСЛОЙНЫМ ВЯЗКОУПРУГИМ ТРУБКАМ ПРИ УСЛОВИИ ЗАКРЕПЛЕНИЯ СТЕНКИ

Исследуется стабилизация пуазейлевского течения жидкости по многослойной трубке при условии закрепления внешней поверхности для различных параметров модели и режимов течения. Проведены численные расчеты инкремента неустойчивости. Показано, что система может быть стабилизирована за счет определенного выбора вязкостей и модулей упругости слоев. Диапазон вычисленных параметров является достаточно широким для стабилизации системы при разных числах Рейнольдса.

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FORMULATION OF THE CONSTRUCTION TOPOLOGICAL OPTIMIZATION PROBLEM FOR RAILWAY STRUCTURES CONSIDERING THE LIMITATIONS ON THE STRENGTH

The main purpose of the paper is the development of the topological structural optimization scientific basis in accordance with the complicated optimization problems of rolling

stock and special railway equipment structures. The theory review and analysis of the current state of structure topological optimization is executed. The classical variation and FE formulation of the topological optimization problem are considered in the paper. The SIMP-method idea and peculiarities of its realization were presented. The problem of stress-constrained structure mass minimization is considered. The problems caused by taking into account the strength limitations were considered in the paper in details. The scientific novelty is the development of the optimal design theory adapted to the rolling stock and special railway equipment structures problems.

Introduction. Topological optimization as the independent scientific research field starts from the paper of talented Australian inventor Michell [16], which was published in 1904. In [16] Michell for the first time obtained the optimal criterion for material distribution in trusses. In 1960 on the second ASCE conference on electronic computation Schmit suggested his revolutionary idea of objects and systems designing with minimal cost due to mathematical programming methods [22]. Schmit idea rapidly enough began to be used in the size and shape structural optimization and later in the topological structural optimization [15].

Numerical mathematical programming methods in topological optimization were intensively investigated from 80-th [3]. Analysis from [20] shows that for the numerical FE topology optimization problems solution the following mathematical programming methods are used:

• gradient methods including the sequential linear programming methods, the sequential quadratic programming methods, the convex linearization methods and method of moving asymptotes (MMA) are the most widely applied [2];

• nongradient methods with two popular algorithm groups: genetic [10] and evolutionary [26];

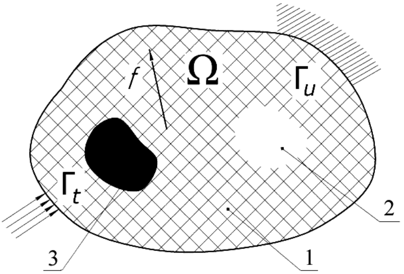
• optimality criteria methods (heuristics methods) [23].

Gradient methods have the most distribution in the modern optimization software (Altair HyperWorks OptiStruct, Dassault Systems Simulia ABAQUS, ANSYS and others) now. MMA proposed by Svanberg [25] can be considered separately from the large quantity of gradient methods since the algorithm of this method has been based on the number of calculation optimization models. The MMA idea is the special type of the convex approximation of the objective function and the strength limit functions [8, 25].

Thus, topological optimization is the conceptual structural design and improvement tool, which requires post-processing and detailed analysis of the obtained results.

Classical formulation of topological optimization problem. The main idea of the structural topological optimization is to obtain the optimal material distribution in the preliminary defined domain. Classical formulation of the problem is the structural pliability minimization (the stiffness maximization) under volume or mass limits [5].

It is considered some design domain Ω (figure 1) in a space R^2 or R^3 which is the part of deformed solid body. In the design domain we define the body



1 – a design point; 2 – a point with no material; 3 – a point with fixed material

Figure 1 – The generalized shape design for the problem of optimal material distribution in a two-dimensional domain

forces *f*, the boundary distributed load Γ_t ($\Gamma_t \subset \Gamma \equiv \partial\Omega$) and the section boundary conditions Γ_u. The design optimization problem can be defined as the problem of finding the optimal value of the stiffness tensor $E_{ijkl}(x)$, which can vary in the Ω domain.

The energy bilinear form (the internal virtual work of an elastic body at the equilibrium *u* and for an arbitrary virtual displacement *v*) can be written as:

$$a(u,v) = \int_{\Omega} E_{ijkl}(x)\varepsilon_{ij}(u)\varepsilon_{kl}(v)d\Omega ,$$

where $\varepsilon_{ij}(u) = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right)$ are the linearized strains; and the load linear form in the following view:

$$l(u) = \int_{\Omega} fu\, d\Omega + \int_{\Gamma_t} tu\, ds ,$$

where the problem of the material pliability minimization (global stiffness maximization) takes the form:

$$\min_{u\in U, E} l(u) ,$$

if $\underset{E\in E_{ad}}{a_E}(u,v) = l(v), \forall v \in U$ – the equilibrium equation in the variation form, where U – the space of kinematically admissible displacement fields; E_{ad} – the set of admissible stiffness tensors.

The index E indicates that the bilinear form a_E depends on the design variables.

Formulation of the FE topological optimization problem. The design domain Ω is divided in N finite elements. The design variable $\rho_e(x) \in (0,1]$, $e = 1, ..., N$ corresponds to each finite element and characterizes the relative material density [4]. These design variables create the vector $\vec{\rho} \in R^N$. The structural global stiffness matrix $\vec{K}(\rho_e) \in R^{d\times d}$ depends on design variables, where d is the number of degrees of freedom. For the external load vector $\vec{f} \in R^d$, the displacement vector is $\vec{u} \in R^d$, and the main equilibrium equation has the following view:

$$\vec{K}(\rho_e)\vec{u} = \vec{f}.$$

Considering the linear elastic features of material, the strain tensor and the stress tensor can be written through the kinematic equation and the state equation respectively:

$$\varepsilon_{ij} = \frac{1}{2}(\vec{u}_{i,j} + \vec{u}_{j,i}); \; \sigma_{ij} = \vec{D}_{ijkl}\varepsilon_{kl},$$

where $\vec{D}$ – the state matrix, which depends on Poisson ratio μ and Young modulus $\vec{E}_0$.

SIMP is the most popular solving method for the structural topological optimization problems nowadays. In the base of SIMP is the conception of **S**olid **I**sotropic **M**icrostructure (or **M**aterial) with **P**enalization. Bendsoe in [4] proposed the fundamental idea of this conception. Firstly the term "SIMP" was proposed by Rozvany in [21] but it was used later.

The SIMP approach implies the replacement of integer variables by continuous variables and obtaining the discrete solution of 0-1 values [4, 5, 8]. It means that the optimal design must have only domains with material – "1" and without material – "0". The intermediate values of the density function $\rho_e(x)$ in the interval (0, 1] should be penalized.

The material properties for each finite element are expressed through the penalty Young modulus $\vec{E}_e$ in the following form:

$$\vec{E}_e = \rho_e^p \vec{E}_0,$$

where $\vec{E}_0$ – Young modulus for solid isotropic material, p – penalty parameter, which must be larger than 1 for the density function penalizing in the $0 < \rho < 1$ interval [4]. In [5] Bendsoe recommends to take penalty parameter p larger or equal 3 ($p \geq 3$) as the intermediate values of the density function in the result optimal design don't arise. Thus, the penalty function in SIMP is realized without any explicit penalty schemes.

When we use the penalty Young modulus $\vec{E}_e$ the global stiffness matrix has explicit dependence on design variables and relative density of each finite element:

$$\vec{K}(\vec{\rho}) = \sum_{e=1}^{N} \rho_e^p k_0,$$

where k_0 – the finite element stiffness matrix for the solid isotropic material Young modulus E_0.

The structure pliability minimization (stiffness maximization) for the given volume or mass is equivalent to the structure deformation energy in the equilibrium state. The FE topological optimization problem formulation has the following view:

$$\min C(u) = \vec{u}^{\mathrm{T}} \vec{K} \vec{u} ;$$

$$\vec{K}\vec{u} = \vec{f};\quad m(\vec{\rho}) = \sum_{e=1}^{N} \rho_e \le m_0;\quad 0 < \rho_{\min} \le \rho_e \le 1,$$

where C – the structure pliability; $\vec{u}$ – the global displacement vector; $\vec{K}$ – the global stiffness matrix; $\vec{f}$ – the global external load vector; m_0 – limit of the structure maximal mass; $\rho_{\min}$ – minimum relative density (usually $O(10^{-3})$ [5]).

Formulation of topological optimization problem considering the limitation on the strength. The topological optimization problem statement for the structure mass minimization with taking into account stress-constrained condition is more realistic then classical statement for minimization of structure pliability. Such formulation has the following view:

$$\min m(\vec{\rho}) = \sum_{e=1}^{N} \rho_e; \tag{1}$$

$$\vec{K}\vec{u} = \vec{f};\quad \frac{F(\sigma_e)}{[\sigma]} \le 1;\quad 0 < \rho_{\min} \le \rho_e \le 1,$$

where $F(\sigma_e)$ – the function that define stress distribution in the design domain finite elements; $[\sigma]$ – the allowable stress value for the given material.

Mises criterion is often used for the calculation of the equivalent stress values σ_{vM} for isotropic materials:

$$\sigma_{vM}^2 = \frac{1}{2}[(\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2] + 3(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2).$$

Peculiarities of problem caused stress limitation. The fact of taking into account stress limitation causes some difficulties for topological optimization problems. Usually structural topological optimization problems have the convergence problems depending on the stress features [14].

The stress singularity problem in structures topological optimization was firstly discovered while solving the truss design problems. In [7] it was shown that n-dimensional space of allowable structure designs has singular subspaces with less then n dimension. Therewith the global optimal design of the structure is often in such singular subspace [11, 14]. Nonlinear programming algorithms cannot identify such domains, so there are only local optimal designs of the structure as a result.

To solve the problem of relaxed singularity limitations on stress it is necessary to remove degenerate subspaces from the space of admissible projects and, as a result, to get a global optimum of the problem by the methods of nonlinear programming. For the topological optimization of frame and truss structures some relaxation methods were used, for example, ε-relaxation method and method of the smooth envelope functions (SEF). Later these methods were adapted to design problems for continuous structures [13].

The idea of ε-relaxation is that the traditional view of stress limitations

$$(\sigma_{vM} - [\sigma])\rho_e \le 0$$

is replaced by changing the lower limit for small value $\varepsilon > 0$. Thus stress condition will have the following view:

$$(\sigma_{vM} - [\sigma])\rho_e \le \varepsilon \ .$$

The ε-relaxation of strength limitations allows to get the relative material density ρ_e with greater than zero sufficiently small values. Thus the singular subspaces are excluded. In [24] it was shown that if the global optimum of the problem can be obtained by using the ε-relaxation, it is not guaranteed that the solution of the initial problem with not relaxed restrictions on the strength will converge to the global optimum.

Allowable stress criterion in structural topological optimization problems. When we consider the stress-limited topological optimization problem the following difficulty is the local character of stress limitations. In the continual formulation of the problem the stress limits must be considered for each material point. In the discrete formulation of the problem (for example FE) the number of material points is finite but still very large for the practical realization. There are several methods for considering the stress limitations in the topological optimization problems.

One of the simplest approaches is the control of the stress values for the given nodes of every finite element. This method is called the local approach and is used in [19]. The local approach requires the large quantity of calculations as we consider that the number of stress limits is comparable with the number of the finite elements. It is possible to reduce the number of the limitations if calculations of the sensitivity are made only for active limitations.

Other approach consists of reduction of all local stress limitations into one global constraint. This method is called the global approach and is used in [9]. As the aggregation function the p-norm function is used:

$$\sigma_{PN} = \left[\sum_{e=1}^{n} \left(\frac{F(\sigma_e)}{[\sigma]} \right)^p \right]^{\frac{1}{p}}, \tag{2}$$

or Kreisselmeier-Steinhauser function (KS-function) [18] is used:

$$\sigma_{KS} = \frac{1}{p} \ln \left[\sum_{e=1}^{n} \exp\left(p \frac{F(\sigma_e)}{[\sigma]} \right) \right]. \tag{3}$$

Both function (8) and (9) are smooth and differentiable. Parameter p controls the level of smoothness.

The disadvantage of global constraints method is significantly worse control over the level of local stresses in comparison with the method of local constraints. The advantages of the global constraints method is the reduction of computational operations performed during the optimal design process.

The third approach implies the grouping of the finite elements into the blocks and using the separate aggregation function for every block. Such method is called the block-aggregate approach and is used in [17].

In this approach the constraints for every block can be written in the following form:

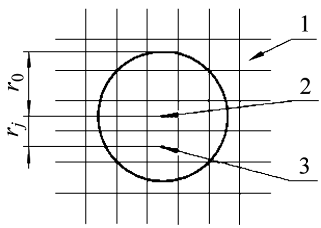
$$\sigma_{\max} = \max\left(\frac{F(\sigma_e)}{[\sigma]}\right). \tag{4}$$

When we use the block-aggregate approach the number of constraints is considerably reduced in comparison with the local approach of the local stress level control. The disadvantage of this approach is that function (11) is non differentiable.

In the end we must pay attention to a the new approach proposed in [11]. This method is called the cluster approach. According to this approach the finite elements which have the comparable stress level are grouped into the clusters by some rule (the stress level technique or the distributed stress technique).

Filtering of design variables. Limits on the stress nonlinearly significantly depend on the project type. The change of the relative density ρ_e in the neighborhood regions changes the stress level. This effect becomes stronger in the critical regions with big stress gradients (the stress concentration), for example in the sharp corners. This problem is called mesh-dependency problem [5, 11, 13]. Thus the topology optimization problem statement and its solution algorithm must exclude the convergence problems.

The density filtering approach [6] was proposed and later proved for the ill-posed topological optimization problem. The filtered density variables ρ_e are created by taking the weighted average of the neighborhood design variables x_j. The design variables filter is written in the following form:



1 – FE-mesh; 2 – e design variable; 3 – j design variable

Figure 2 – The visualization of design variables filter

$$\rho_e(\vec{x}) = \frac{\sum\limits_{j\in\Omega_e} w_j x_j}{\sum\limits_{j\in\Omega_e} w_j},$$

where Ω_e – the domain (for e finite element) consisted of all elements j which lie in the circle of r_0 radius and measured between gravity centers of the neighborhood elements (Figure 2); w_j – average weighted coefficient.

The average weighted factor is obtained according with the following form [6]:

$$w_j = \frac{r_0 - r_j}{r_0}.$$

Note that the weight is equal to zero for all variables which are outside the domain Ω_e. From the view of method realization, the weighted matrix $\vec{W}$ is entered into the relative material density function in the following form:

$$\rho_e(\vec{x}) = \sum_{j=1}^{n_e} W_{el} x_j .$$

Formulation of topological optimization problem for the rolling stock and special railway equipment structures considering strength constraints. The creation of strength-constrained structures with minimal mass is important for the railway engineering industry. The classical equations for topological optimization problem are unacceptable for the rolling stock and special railway equipment problems because the creation of the most rigid structure with only volume or weight constraints is not expedient.

The structure strength of railway vehicles is estimated by two criterions: the allowable stress criterion and the fatigue strength safety factor criterion [1]. The strength is estimated on the design stage and on the stage of prototype testing.

We later consider the stress-constrained structural topological optimization problem statement which equivalent to the problem with the allowable stress criterion. The problem of structure fatigue strength estimating hasn't been investigated enough yet. The above literature review showed that there is a very small number of papers devoted to the problem of structure topological optimization applied to the rolling stock and special railway equipment structures. Thus, the problem of structure topological optimization for rolling stock and special equipment of railways based on the strength of complex constraints is a relevant scientific and technical issue nowadays. The complex limitation on the structure strength includes the allowable stress criterion and the fatigue strength safety factor criterion.

If the structure is under the action of the non-central fatigue cycle then the structure normal stress σ_a less the mean stress σ_m. The number of fatigue cycles by the structure life time is $N > N_G$, where $N_G = 2 \cdot 10^6$ cycles – the abscissa of the fatigue plot breaking point. Non-central fatigue cycle can be transformed into the fully symmetric cycle form. Such symmetric cycle becomes equivalent to the non-central fatigue cycle under breaking action. The peak stress of the fully symmetric cycle is:

$$\sigma_{ae} = \sigma_a + \psi_{\sigma D} \sigma_m = \sigma_a + \frac{\psi_\sigma}{K} \sigma_m , \qquad (5)$$

where K – the endurance limit reduction coefficient. This coefficient takes into account such factors acting on the endurance strength as stress concentration, the scale factor, the surface quality, the operation factor, the technological strengthening methods; $\psi_{\sigma D} = \dfrac{\psi_\sigma}{K}$ – the asymmetry cycle influence coefficient for the structure with actual dimensions and stress concentrations; ψ_σ – the asymmetry cycle influence coefficient for the smooth laboratory pattern:

$$\psi_\sigma = \frac{2\sigma_{-1} - \sigma_0}{\sigma_0},$$

σ_{-1}, σ_0 – the endurance limits for the smooth laboratory pattern with the fully symmetric load cycle and the pulsating load cycle respectively.

The calculations for the pattern under the fatigue strength safety factor by the stresses is executed by the following formula:

$$n_\sigma = \frac{[\sigma]}{\sigma_e} \geq [n], \tag{6}$$

where $[\sigma]$ – the allowable stress; σ_e – the equivalent operating stress obtained by the hypothesis of strength.

The allowable stress $[\sigma_{-1}]$ is the endurance limit for the symmetric cycle in this case:

$$[\sigma] = [\sigma_{-1}] = \frac{\sigma_{-1}}{K}. \tag{7}$$

Thus the fatigue strength safety factor under the linear stress mode can be defined by equations (5) and (7) included in (6):

$$n_\sigma = \frac{[\sigma_{-1}]}{\sigma_{ae}} = \frac{\dfrac{\sigma_{-1}}{K}}{\sigma_a + \dfrac{\psi_\sigma}{K}\sigma_m} = \frac{\sigma_{-1}}{K\sigma_a + \psi_\sigma \sigma_m}. \tag{8}$$

The expression (8) for the fatigue strength safety factor was proposed in 1940^{th} by S. V. Serensen and R. S. Kinasoshvili. This expression was widely used in the industry particularly in the railway industry. Today in the railway industry the formula (8) is the base expression for the fatigue strength estimate when we don't have the peak stress distribution bar chart and the material endurance curve parameters.

The mean cycle stress σ_m in (8) is defined by the stresses under the static load action and quasistatic forces in the tractive, braking modes and movement in curves:

$$\sigma_m = \sigma_{st} + \sigma_{tr/br} + \sigma_{curv},$$

where σ_{st} – the stress under vertical static load action; $\sigma_{tr/br}$ – the stress caused by tractive or braking load action; σ_{curv} – the stress by movement in curves.

The peak cycle stress σ_a for the railway vehicles is

$$\sigma_a = K_{dv}\sigma_m . \tag{9}$$

where K_{dv} – the vertical dynamics factor obtained by the dynamics test results.

After substituting of (9) into (8) the following strength condition is obtained:

$$\frac{\sigma_{-1}}{[n]\sigma_m(K_{dv}K + \psi_\sigma)} \geq 1 .$$

Let us make the following substitute

$$\frac{\sigma_{-1}}{[n](K_{dv}K + \psi_\sigma)} = [\sigma] . \tag{10}$$

Thus the fatigue strength safety factor strength condition is transformed into the traditional view of the allowable stress condition:

$$\frac{[\sigma]}{\sigma_e} \geq 1 . \tag{11}$$

The peculiarity of the obtain equation (11) is that such facts as the allowable safety factor of the fatigue strength $[n]$, the asymmetry cycle influence coefficient ψ_σ, the endurance limit reduction coefficient K and the vertical dynamics factor K_{db} are taken into consideration.

The allowable safety factor of the fatigue strength $[n]$ is equal to 2,0 for the rolling stock and special railway equipment. The asymmetry cycle influence coefficient $\psi_\sigma = 0{,}3$ is for $\sigma_m > 0$.

The endurance limit reduction coefficient can be calculated by the following expression:

$$K = \frac{K_1 K_2}{\gamma m}\beta_K , \tag{12}$$

where K_1 – the coefficient of the material heterogeneity influence; K_2 – the influence coefficient of the internal stresses; γ – the coefficient of the scale factor influence; m – the coefficient of the surface interaction; β_K – the effective strength reduction factor.

For example, for the molded parts with the lateral dimension near 300 mm $\gamma = 0{,}55$, $K_1 = 1{,}3$, $K_2 = 1{,}0$. The coefficient m of the surface interaction is equal to 0,85 if the operation of machine is rough. For the cored beam with the rectangular cross section β_K is equal to 1,0. So the calculated from the equation (12) endurance limit reduction coefficient K is equal to 2,78.

The endurance limit σ_{-1} for the smooth laboratory pattern made from the Steel 09G2 GOST 19281-89 under the fully symmetric load cycle is equal to 240 MPa. The normative value of the vertical dynamics factor K_{dv} for the railway vehicles is equal to 0,35 for the first suspension level. Thus the fatigue strength of the structure is ensured in the course of N cycles for the railway vehicles if the allowable stress is $[\sigma] = 94{,}3$ MPa in accordance with equation (10).

The insertion of the fatigue strength criterion in the topological optimization problem does not change the problem formulation as this criterion is reduced to the allowable stress constraint.

The feature of this complex strength-constrained problem statement is the calculating the allowable stress according to (10).

Thus the formulation of the topological optimization problem for the rolling stock and special railway equipment with taking into account strength limitations has the similar to (1) view:

$$\min\ m(\vec{\rho}) = \sum_{e=1}^{N} \rho_e$$

under conditions of

$$\vec{K}\vec{u} = \vec{f}\ ,\ \frac{F(\sigma_e)}{[\sigma]} \le 1\ ,\ 0 < \rho_{\min} \le \rho_e \le 1\ ,$$

where $[\sigma]$ – the allowable stress for the case of the structure fatigue strength ensuring under N cycles:

$$[\sigma] = \frac{\sigma_{-1}}{[n](K_{dv}K + \psi_\sigma)}\ .$$

Conclusions. There were analyzed the main historical steps of creating the structural topological optimization theory. The classical variation and FE structural topological problem statements are presented in the paper. The features of SIMP method concept and realization are considered.

In the paper we obtained the equation for the stress-constrained topological optimization problem in the mass minimization form. Particularly the singular problem definition and approaches for problem solving are considered. The mesh-dependency problem and the filtering of design variables are considered in the paper. The methods of the stress limitations inserting in the equations for the topological optimization problem are considered.

Thus the review and analysis of the current state of the structural topological designing showed that this field of investigations had been given actively developed at the last time and it is of great interest today.

There were used such modern design tools as topological optimization for the problems of the rolling stock and special railway equipment structures creating and improvement which are relevant problems nowadays.

In the paper the statement for complex strength-constrained topological optimization problem for the rolling stock and special railway equipment structures was proposed. It includes the allowable stress criterion and the fatigue strength safety factor criterion.

In the paper we show that the fatigue strength safety factor criterion can be reduced to the allowable stress criterion under the stipulation that the allowable stress is chosen special.

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Б. М. ТОВТ

ПОСТАНОВКА ЗАДАЧИ ТОПОЛОГИЧЕСКОЙ ОПТИМИЗАЦИИ ЖЕЛЕЗНОДОРОЖНЫХ КОНСТРУКЦИЙ С УЧЕТОМ ОГРАНИЧЕНИЙ НА ПРОЧНОСТЬ

Главная цель статьи заключается в развитии научных основ теории топологической оптимизации конструкций в части решения сложных задач усовершенствования конструкций подвижного состава и специальной техники железных дорог. Выполнен обзор теорий и анализ современного состояния методов топологической оптимизации конструкций. Приведены классическая вариационная и конечно-элементная постановки задач топологической оптимизации. Рассмотрена идея и особенности реализации SIMP-метода для их решения. Приведена постановка задачи топологической оптимизации в виде минимизации массы конструкции с учетом ограничений по напряжениям. Детально рассмотрен ряд проблем, возникающих при введении подобных ограничений в задачу оптимизации. Научная новизна заключается в постановке задач топологической оптимизации, адаптированной к решению задач о проектировании конструкций подвижного состава и специальной техники железных дорог.

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