UDC 620.178

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NEW SECTION OF MECHANICS

Fatigue is a stern scourge of modern machinery; friction is an amazing phenomenon of nature; wear is a crafty enemy of any system which moves or is deformed. Tribo-Fatigue is the unifying science that treats fatigue, friction and wear.

In this paper presented a brief review of some studies lying on the path from Mechanics of fatigue fracture and Tribology to Tribo-Fatigue, new and perspective section of mechanics.

Introduction. The aim of any *scientific discipline* is to perceive and describe particular regularities and peculiarities of the evolution of some phenomena, situations, and events stemming from the existence of real or thinkable events that possess specific features [1]. Proceeding from the idea that the perception of a new subject gives, as a rule, birth to a novel scientific discipline, we may, as applied to mechanics, construct the hierarchy of the subjects associated with solid mechanics [1–3]. Figure 1 shows its scheme in a very simplified manner. The aim of constructing such a hierarchy is to identify the place of Mechanical Fatigue [1–10], Tribology and Tribo-Fatigue [11–26 *etc.*] in the system of established scientific disciplines and characterize their role in this system.

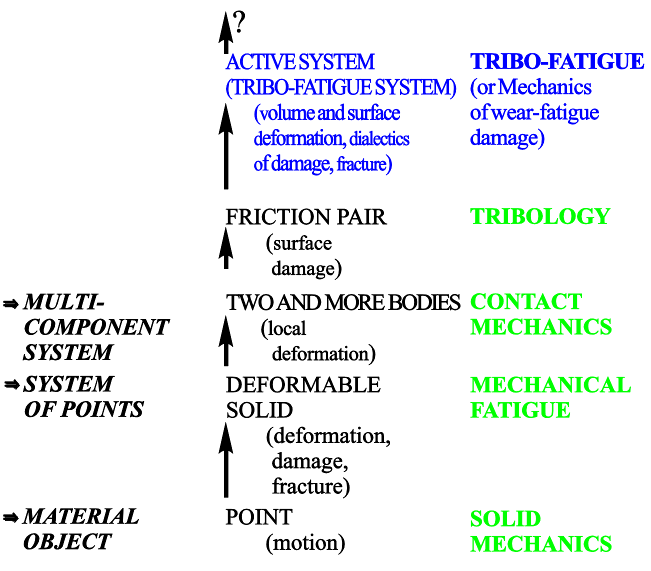

Figure 1 – Simplified hierarchic structure of some objects studied by mechanics: from simple to complex

When a material object was represented mentally as a dimensionless *point* having no structure and possessing only a capability of moving in the space and time along any trajectory and in any direction, need arose for *Theoretical Mechanics* in order to understand and describe the motion of such physically unreal object. The idea of "the scale of a point" made theoretical mechanics a useful science since it became possible to analyze correctly the motion of points-planets or points-electrons – enormously huge objects of the universe and unimaginably small objects in the microcosm. Now, if one assigns to these "great points" a mass, for example, then the laws of interaction between celestial bodies during their motion can be established. The diverse array of motion associated with *Mechanics of Space Flight*, *Mechanics of Mechanisms and Machines* can be analyzed by methods of theoretical mechanics.

A set of points connected in a certain manner is a *continuum*. One sort of it is a *solid*, which possesses specific features, e.g., *rigidity* and *strength*. When it was found that points in a solid are capable of moving or shifting relative to each other upon subjecting them to an external load, the concept of a new subject – *the deformable solid* – had to be developed. It is natural that *Deformable Solid Mechanics* had to be established in order to study the stress-strain state at any point of the solid and to understand and describe regularities and peculiarities of variations in sizes and shape deflections of the solid as the whole. The deformable solid was simply called a *material* or a *specimen*, or a *structural element* depending on the specific aim of the study. Specific properties of such subjects are studied in various domains of mechanics as *Mechanics of Materials*, *Structures*, *Composites*, *Soil* etc.; *Mechanics of Damage* and *Fracture Mechanics* (under static, impact, cyclic and other modes of loading); *Mesomechanics*, *Micromechanics* and others. In these cases regularities, peculiarities and consequences of the reversible (*Elasticity Theory*) and irreversible (*Plasticity Theory*) movement of points of deformable solids are studied. Deformable solids are treated as having a variety of specific properties, e.g. *viscoelasticity*, *elastoviscoplasticity* and others. Thus, solid mechanics has become one of the most powerful tools to study the behavior of objects under different operating and testing conditions. The application of computers and the development of computer simulation methods substantially increased the efficiency of calculations and gave rise to *Computational Mechanics*.

Over time solid mechanics research of deformation and fracture of specimens and structural elements under the influence of re-variable loads became important. This was because the sudden failures of the most critical parts of machines lead not only to substantial material damage but also to a breach of security guarantees for the people. And then a scientific discipline that was called *Mechanics of fatigue fracture* or simply *Fatigue* emerged.

A deformable solid is only one component of numerous and diverse *mechanical systems*. The simplest case of the *compression of two stationary solids* gave birth to a new approach in the elasticity theory, called a *contact problem*. It became the origin of *mechanics of contact of bodies* (components) under static, impact, cyclic or other modes of loading.

The next subject deals with a *friction pair* whose main feature is the *relative sliding motion between two solids* compressed externally by load. Thus, a special scientific discipline known as *Tribology* was born whose main objectives was to study regularities and peculiarities of friction and surface damage of materials at sliding, rolling, slippage, impact etc. [27–38]. In essence, any friction pair is a *multicomponent system* since the so-called *third body* inevitably emerges from it [39, 40]. The third body is formed in the zone of movable contact from a lubricant and/or products of "tribo-destruction" of thin surface layers of the contacting bodies.

A peculiar subject, which is even more complex than a friction pair, is known as an *active system*. The concept of the active system has been introduced quite recently (at the end of the 20^{th} century) [41, 42]. The active system is a mechanical system which *carries the operating cyclic load and transmits it* in a manner *in which the friction process occurs simultaneously at any* of *its manifestation* [43–45]. In other words, the active system is a friction pair in which at least one member undergoes repeated volumetric deformation. These systems experience *complex wear-fatigue damage* (WFD) due to the kinetic effect of several phenomena associated with mechanical fatigue, friction, wear, erosion, corrosion, etc. [18, 46–50]. For this reason, active systems are also called *Tribo-Fatigue systems* (TFS) [24, 51, 52]. The associated science dealing with this subject is called *Tribo-Fatigue* [11, 15, 18, 19, 21–23].

Thus, *Tribo-Fatigue is the science concerned with the study of the complex wear-fatigue damage and fracture of active (Tribo-Fatigue) systems* [45, 53]. Within this context, the term *Tribo-Fatigue* symbolizes modern Tribology including Tribological Reliability as well as Fatigue Damage including Strength Reliability. If one considers physical-mechanical processes determined by these notions, their *dialectic interaction and mutual influence* should be born in mind in the following manner:

$$\text{Friction (and wear)} \Leftrightarrow \text{Fatigue} = \text{Tribo-Fatigue.}$$

Since Tribo-Fatigue is a new branch of mechanics [1, 19, 23, 45, 54], it can also be determined as the *mechanics of Tribo-Fatigue systems* [24].

It is necessary to specify in fundamental nature of concept of fatigue at a volume stressing and a friction. Authors of modern books treat *mechanical fatigue* as the process of the gradual accumulation of damages of a material under the effect of alternating stresses or deformations that causes changes in its structure and properties, the nucleation and propagation of cracks and fracture. "So, the term *fatigue of metals* means the behavior of metals that undergo repeated stresses", – that is the *general meaning* of the notion under consideration, which is "far from ideal and too indefinite and unclear", though "generally accepted" and "widely used" – it was given by H. J. Gough in 1926 [10].

Here the *material behavior* may mean whatever one likes – damage, cracks, or fracture etc., *regardless of physical mechanisms* that are discovered sometimes under certain conditions of loading of one or another material, including different scale levels, viz. atomic, submicro-, micro-, meso-, and macrolevels. The Gough's generalized definition remained applicable even when I. V. Kragelskii established the fatigue wear mechanism at sliding (1939). During the next decades *volume* or *mechanical fatigue* and *surface* or *frictional fatigue* were distinguished. The new adjectives used with the "old" term changed nothing in its meaning; they only concretized conditions of origination and the zone where fatigue processes occur. And, finally, when *mechanics of complex* or *wear-fatigue damage* (*Tribo-Fatigue*) emerged at the end of the last century, the old term *fatigue* arrived again on the scene and successfully claimed.

Table 1 presents the comparative analysis of methods of the study and calculation of the objects investigated in Tribology, Fatigue Fracture Mechanics and Tribo-Fatigue [23, 55, 56].

Table 1 – **Methods of studying objects in three adjacent disciplines**

Discipline	**Subject of study**	**Basic methods of studies**		**Type of damage**
		experimental	**theoretical**	
T (Tribology)	Friction pair	Tribotesting	Contact mechanics	Surface damage (wear, pitting, etc.)
F (Fatigue Strength)	Structural element	Fatigue tests	Mechanics of deformation and fracture	Volume (fatigue) fracture
TF (Tribo-Fatigue)	Tribo-Fatigue system	Wear-fatigue tests	Mechanics of complex wear-fatigue damage and fracture	Complex surface damage and volume fracture

In the eighties of the XX century the scientific basis of Tribo-Fatigue, both in terms of experimental and theoretical development, has received much attention with over 800 papers over the first 30 years of its establishment [1, 15, 17, 23, 25, 26, 52, 56–67]; six *International symposia on Tribo-Fatigue* have been organized during the last 15 years [26, 68–73].

As it is known any review is subjective due to a natural reason: it is written by concrete people. But we hope that while being concise our review is sufficiently informative with the following limitation: physical mechanisms of damage, fatigue and fracture are not analyzed. Here only mechanics of wear-fatigue damage and fracture is discussed.

Concept of Direct and Back Effects. Let us turn our attention now to two important effects in Tribo-Fatigue: *direct effect (DE)* is the influence of conditions and processes involving friction and wear on variations in the fatigue strength characteristics (Figure 2), also known as the Sosnovskiy – Serensen effect [1]; *back effect (BE)* is the influence of conditions and processes of fatigue damage due to cyclic deformation on variations in the friction and wear characteristics (the wear resistance parameters) (Figure 3), also known as the Sosnovskiy – Sharai effect [1, 74].The analysis of effects provide new results for both Mechanics of fatigue fracture (Figure 2) and Tribology (Figure 3).

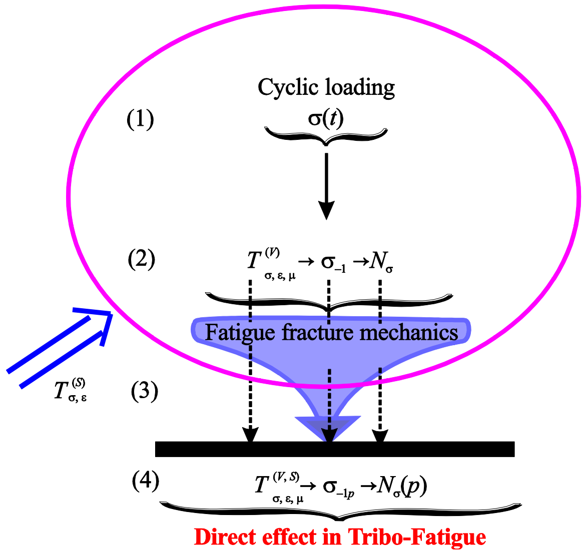

Figure 2 – Direct effect: influence of field of surface(contact) deformation (tensor $T^{(S)}_{\sigma,\varepsilon}$) on variations in fatigue resistance characteristics (σ_{-1} is the fatigue limit, N_σ is the fatigue durability, $T^{(V)}_{\sigma,\varepsilon,\mu}$ is the stress (strain) tensor at volume (cyclic) loading)

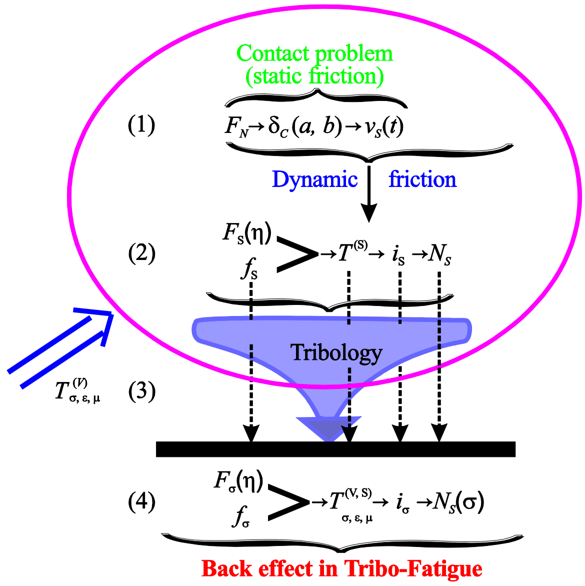

Figure 3 – Back effect: influence of field of volume deformation (tensor $T^{(V)}_{\sigma,\varepsilon,\mu}$) on variations in friction, wear, and life characteristics (F_S, f_S are the friction force and coefficient, *i* is the wear, N_S is the wear life, $T^{(S)}_{\sigma,\varepsilon}$ is the stress (strain) tensor at contact interaction)

The combination of both effects can be described as follows

$$\begin{cases} \left.\begin{matrix} F_\sigma(\eta) \\ f_\sigma \end{matrix}\right\rangle \to T^{(V,S)}_{\sigma,\varepsilon,\mu} \to i_\sigma \to N_S(\sigma); \\ T^{(V,S)}_{\sigma,\varepsilon,\mu} \to \sigma_{-1p} \to N_\sigma(p), \end{cases} \quad (1)$$

where represents the *combined stress (strain) tensor* from the contact and off-contact loads. These effects occur simultaneously and in one and the same zone of deformation called the *dangerous volume* in a Tribo-Fatigue system; σ_{-1p} and $N_\sigma(p)$ are the fatigue limit and the fatigue durability of the material that account for the influence of the friction and wear conditions (the index *p*); $F_\sigma(\eta)$ and f_σ are the friction force and friction coefficient, respectively. These parameters are determined with the consideration of the cyclic loading conditions (the index σ); i_σ, $N_S(\sigma)$ the wear and durability according to the criterion of wear resistance with account for the influence of stresses σ due to off-contact loads.

The analysis of direct and back effects with the concrete examples is given further.

Thus, the evolution from *Mechanics of fatigue fracture* and *Tribology to Tribo-Fatigue* involves two progressive generalizations [55]. The first generalization is the unified field of Tribology which evolved upon the recognition that friction, wear, and lubrication *simultaneously* affect the performance of contacting surfaces in machine elements. Further recognition that simultaneous effect of fatigue processes involved must also be account for led to the establishment of the *Tribo-Fatigue*, the second unifying generalization.

Typical Tribo-Fatigue Systems and Conditions of their Operation. *The simplest types of Tribo-Fatigue systems* are shown in Figures 4 and 5. Complex wear-fatigue damage occurs in these systems as a result of the kinetic interaction involving fatigue, friction, wear, and (or) erosion. *The classification of the basic types of such damages* is given in Figure 6. Table 2 presents the definitions of the mentioned types of wear-fatigue damage for *typical real systems*. Six terms are applied to describe briefly wear-fatigue damage among which are four novel terms [1, 18, 41, 42, 44–46, 49], namely, mechano-sliding fatigue (MSF), mechano-rolling fatigue (MRF), mechano-erosion fatigue (MEF), and corrosion-erosion fatigue (CEF). The terms fretting fatigue (FF) and mechano-corrosion fatigue (MCF) are well-known for many years [47, 50].

According to Figures 4 and 5 and Table 2, complex wear-fatigue damage of Tribo-Fatigue systems is caused by the following *typical effects* [51, 75]:

a) *contact load* (F_N), it is characterized, as a first approximation, by the *contact pressure* $p_a = f(F_N)$ and (or) the *friction stresses* $\tau_w = f(p_a)$, where *f* is the friction coefficient;

b) *alternating (off-contact) load* (*M*), it is characterized, as a first approximation, by the *cyclic stresses* $\sigma = f(M)$;

c) *thermodynamic load*, it is characterized integrally by the *temperature* T_Σ generated by all heat sources; and

d) *electrochemical load*, it is characterized indirectly by the *corrosion parameter* (*D*); note that *stress corrosion* (D_σ), *friction corrosion* (D_τ) and *thermal corrosion* (D_T) should be distinguished.

Figures 4 and 5 and Table 2 present *three basic classes of Tribo-Fatigue systems*: (1) *solid/solid*; (2) *solid/liquid*; (3) *solid/particles*. Here we note that the solid in classes (2) and (3) is not a traditionally considered rigid barrier but a barrier deformed by alternating load. In other words, it is a deformable solid just as in case of class (1) of Tribo-Fatigue systems. Two sources of friction forces are usually found in such systems: 1) as the shaft rotates, a *common (unidirectional) friction force* of sliding F_S or rolling F_R appear (see Figures 4, *a*, *b*); 2) at small reciprocal displacements (deformations) of a shaft under effect of alternating (volumetric) load a *cyclic friction force* $\pm F_c$ is excited (see Figures 4 and 5, *a*). The vectors $\boldsymbol{F_S}$ and $\boldsymbol{F_R}$ are orthogonal to the vector $\boldsymbol{F_c}$.

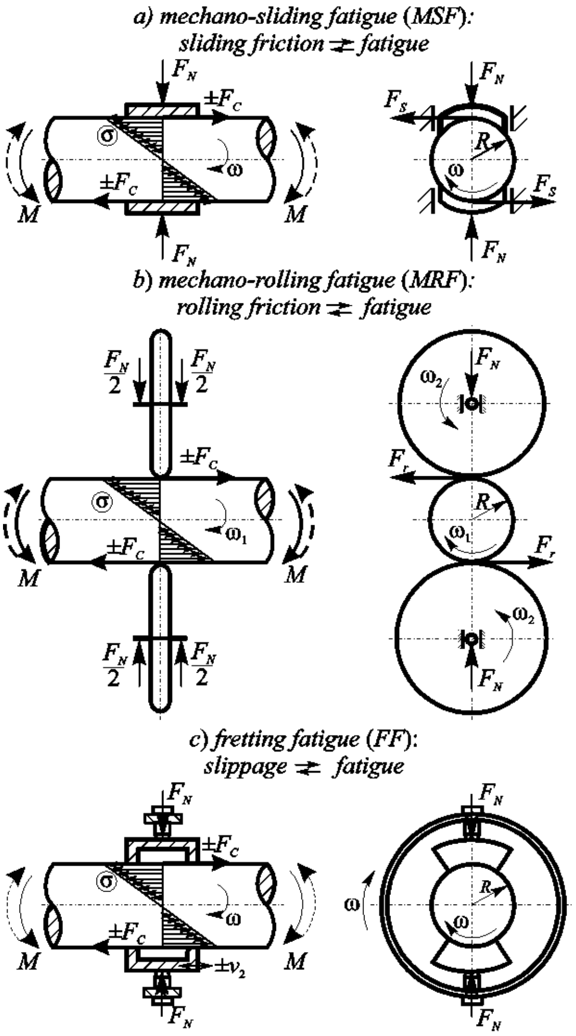

Figure 4 – Simplest principal schemes of typical Tribo-Fatigue systems *solid/solid* in whose working zones characteristic wear-fatigue damages occur

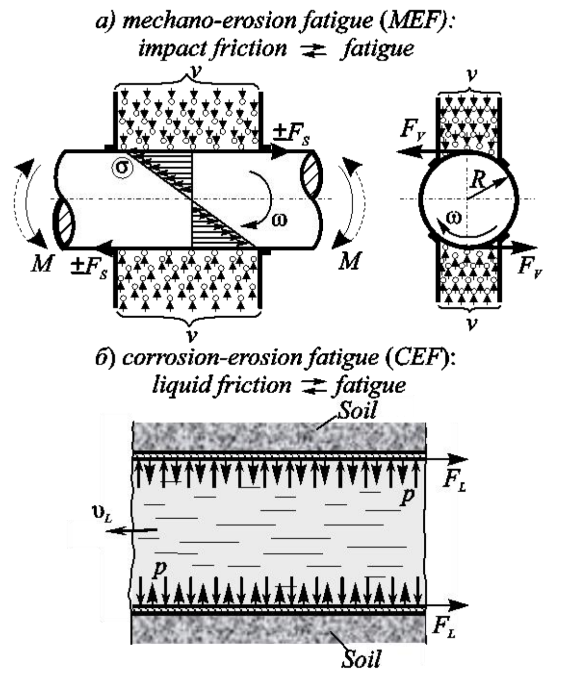

Figure 5 –Simplest schemes of typical Tribo-Fatigue systems *solid/particles* (*a*) and *solid/liquid* (*b*) in whose working zones characteristic wear-fatigue damages occur

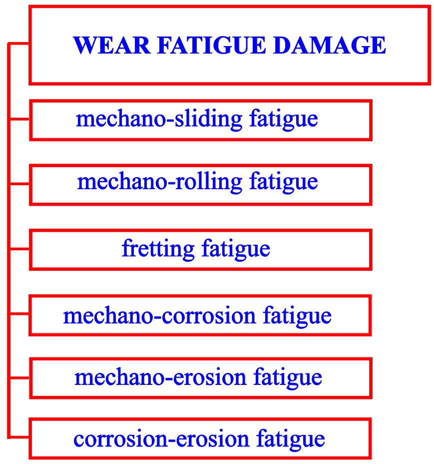

Figure 6 – Classification of main types of wear-fatigue damage according to GOST 30638–99

In general, Tribo-Fatigue systems are much diverse than shown in Figure 4. Figure 7 illustrates how Tribo-Fatigue systems may be formed in a specific way. In the roller-ring and ring-ring systems force F_N is simultaneously responsible for both contact and bending force. Such a situation occurs, for example, in a real wheel-rail system. Only the problems of type A are solved traditionally (for a *roller*/*roller* system). Problems of type B (the *roller*/*ring* system) and C (the *ring*/*ring* system) demand accounting for volumetric deformation of one or both bodies while solving contact problems. The contacting bodies may have a "sandwich" structure consisting of two and more layers (of thickness h', etc.) whose materials may differ in their physical-mechanical properties. Note that in rolling contact the members can have both positive and negative radius of curvature, and different types of the real contact area (rectangular, circular, or elliptical) are realized.

The systematization of various Tribo-Fatigue systems like *solid/solid* is presented in more detail in [76].

In Tribology, and in Mechanics of fatigue fracture usually studied only three mechanical *states of systems* [24, 25, 62, 77–84]: (1) the *stress-strain state*; (2) the *damage state*; and (3) the *limiting state*. In addition, in Tribo-Fatigue studied (4) *translimiting states* [1, 85], reliability states, entropy and information states [25] as well as the safety / risk state [85]. The following is an analysis only (1)–(3) states.

Stress-Strain State. Figure 8, *a* shows a general scheme of the *solid/solid* Tribo-Fatigue system in which the contact load (F_N) and the off-contact loads (M_j, Q_i) act independently but simultaneously. Traditional analysis methodology of this system involves examination of the stress-strain state of either the corresponding friction pair (roller/roller, Figure 8, *c*) or the separate structure member (e.g., a shaft, Figure 8, *b*). The elasticity theory [86–89] is used to determine the stress and strain tensors (Figure 9, *a*):

$$T_{ij}^{(V)} = \sigma_{ij}^{(b)}, i, j = 1,\ \ 2,\ \ 3, \qquad (2)$$

$$E_{ij}^{(V)} = \varepsilon_{ij}^{(b)}, \qquad (3)$$

which are due to the volumetric (the superscripts V and b) loads (in general cases these are the loads of spatial bending, tension-compression and torsion).

Table 2 –**Typical Tribo-Fatigue systems and their complex damage**

Typical Tribo-Fatigue system	Complex damage and fracture	Definition
Crankpin / connecting-rod end with sliding bearing	Mechano-sliding fatigue	Wear-fatigue damage due to the effect of kinetic interaction between the phenomena of mechanical fatigue and sliding friction
Wheel / rail	Mechano-rolling fatigue	Wear-fatigue damage due to the effect of kinetic interaction between the phenomena of mechanical fatigue and rolling friction (rolling friction with slippage)
Spline shaft / bushing	Fretting fatigue	Wear-fatigue damage due to the effect of kinetic interaction between the phenomena of mechanical fatigue and fretting
Propeller shaft / sea water	Mechano-corrosion fatigue	Fatigue of the material under the simultaneous effect of alternating stresses and corrosive environment
Turbine blades / fluid or gas flow carrying solid particles	Mechano-erosion fatigue	Wear-fatigue damage due to the effect of kinetic interaction between the phenomena of mechanical fatigue and erosion
Pipe / fluid flow under pressure	Corrosion-erosion fatigue	Wear-fatigue damage due to the effect of kinetic interaction between the phenomena of mechanical fatigue, corrosion and erosion

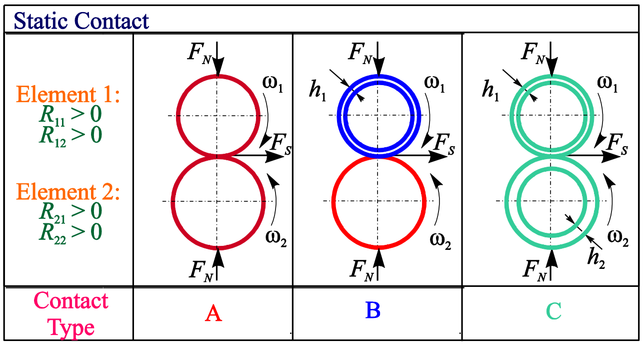

Figure 7 – Classification of contact problems for solids of revolution by conditions of deformation

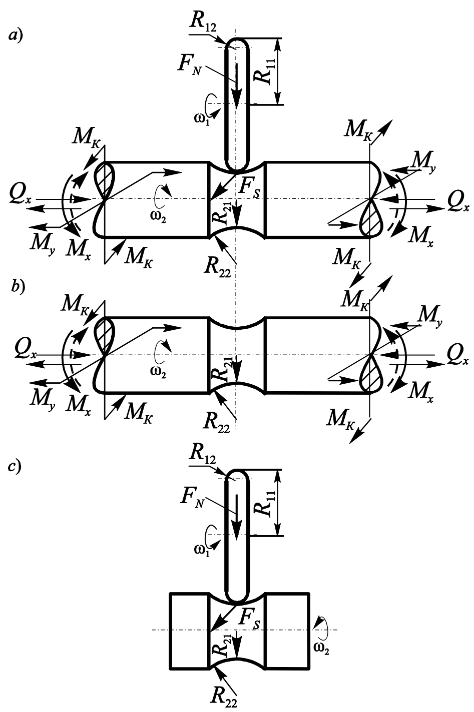

Figure 8 – General structural diagram of *roller/shaft* Tribo-Fatigue system with main rotational motion (ω_1) under independent action of contact (F_N) and off-contact (M, M_K, Q) loads (*a*) and particular schemes of corresponding basic member of system – shaft (*b*) and *roller/roller* friction pair (*c*)

In calculations of a friction pair, in cases of nonconforming movable contact, methods of contact mechanics [90–93] are applied to determine the stress and strain tensors:

$$T_{ij}^{(W)} = \sigma_{ij}^{(n)} + \sigma_{ij}^{(\tau)}; \tag{4}$$

$$E_{ij}^{(W)} = \varepsilon_{ij}^{(n)} + \varepsilon_{ij}^{(\tau)}, \tag{5}$$

which are due to the distribution of the forces normal $p(x, y)$ (the superscript n) and tangential $q(x, y)$ (the superscript τ) to the contact surface (Figure 9, *b*). The tensors T and E with the superscript V are due to the effect of volumetric loads (general cases of three-dimensional bending, torsion and tension-compression) and the tensors with the superscript W are due to the contact interaction of the system members. Solutions (2)–(5) can be found in well-known papers (see, e.g., [86–99] and many others).

When calculating Tribo-Fatigue systems one should determine the combined stress and strain tensors

$$T_{ij}^{(V,\,W)} = \sigma_{ij}^{(b)} + \sigma_{ij}^{(n)} + \sigma_{ij}^{(\tau)}, \tag{6}$$

$$E_{ij}^{(V,\,W)} = \varepsilon_{ij}^{(b)} + \varepsilon_{ij}^{(n)} + \varepsilon_{ij}^{(\tau)}, \tag{7}$$

which are due to the effect of all the loads (Figure 9, *c*), i.e. the contact and off-contact loads (the superscripts V and W). Thus, combined tensors (6), (7) are determined by the superposition of the stresses (strains) caused by both the contact load (tensors (4) and (5)) and the off-contact loads (tensors (2) and (3)).

Here we note two problems arising in solution of equations (6) and (7). First, the study of the stress-strain state in the contact zone is usually restricted by the determination of components of stresses at points of the Z axis and at some points of the contact surface. Clearly, the determination of all components of the stress at any point of a half-space is an arduous task because of great complexity of the functions being integrated. Second, no boundary conditions for such complex problems as (6) and (7) are presented in publications [1, 24, 25].

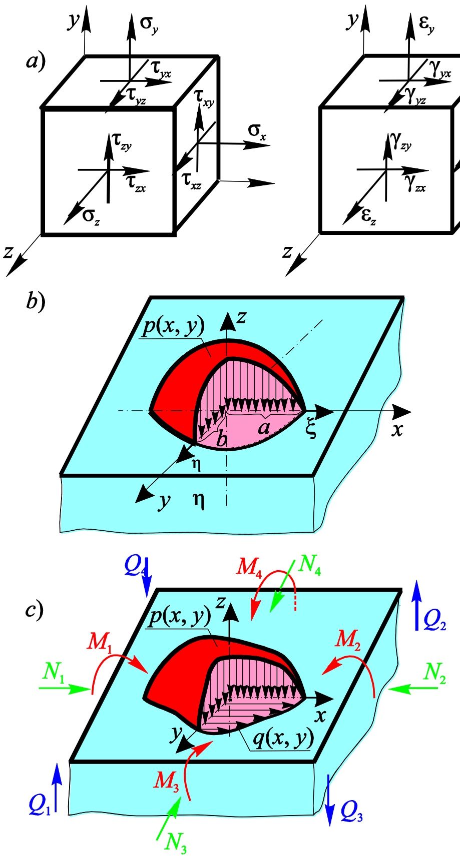

Figure 9 –For analysis of stress-strain state: *a* – structure member (see Figure 8, *b*); *b* – friction pair (see Figure 8, *c*); *c* – Tribo-Fatigue system (see Figure 8, *a*)

Consider the simplest approach to concretization of formulas and (6), (7). Contact between two bodies of some form is usually reduced to the distribution of tractions over the contact area on the half-space according to Hertzian theory (see for example [31, 90, 91, 94]). Then stresses at any point $M(x, y, z)$ of the half-space upon the action of normal contact tractions $p(x, y)$ on area S are calculated (typically, by means of numerical methods) with the use of the influence functions $G_{ij}^{(B)}$ from the fundamental Boussinesq solution $\sigma_{ij}^{(B)}$ [91, 94] for the action of the focused normal force on the half-space

$$\sigma_{ij}^{(n)}(x,y,z)=\iint\limits_{S(\xi,\eta)} p(\xi,\eta)G_{ij}^{(B)}(\xi-x,\eta-y,z)d\xi d\eta. \quad (8)$$

Stresses at any point $M(x, y, z)$ of the half-space upon the action of tangential contact tractions $q(x, y)$ on area S are calculated similarly with the use of the influence functions $G_{ij}^{(C)}$ from the fundamental Cerruti solution $\sigma_{ij}^{(C)}$ [91, 94] for the action of the concentrated tangential force on the half-space

$$\sigma_{ij}^{(\tau)}(x,y,z)=\iint\limits_{S(\xi,\eta)} q(\xi,\eta)G_{ij}^{(C)}(\xi-x,\eta-y,z)d\xi d\eta. \quad (9)$$

Hence stress state in case of contact with friction according to expression **(**4) is the superposition (8) and (9):

$$\sigma_{ij}=\sigma_{ij}^{(n)}+\sigma_{ij}^{(\tau)}. \quad (10)$$

Stresses **(**2) caused by off-contact loads could be determined using the appropriate theory for the concrete case of volume deformation (for example, during bending moment *M*) [87]

$$\sigma_{ij}^{(b)}=\sigma_{ij}^{(M)}+\sigma_{ij}^{(N)}+\sigma_{ij}^{(Q)}, \quad (11)$$

where indices *M*, *N*, and *Q* correspond to the internal moment, the internal longitudinal and transversal forces.

Expressions (8)–(11) allow formulating a mechanical-mathematical *problem on the combined stress-strain state of a Tribo-Fatigue system* in the form of *Sosnovskiy – Sherbakov – Zhuravkov model* [24, 1, 61, 62, 100]

$$\begin{aligned}\sigma_{ij}&=\sigma_{ij}^{(n)}+\sigma_{ij}^{(\tau)}+\sigma_{ij}^{(b)}=\\&=\iint\limits_{S(\xi,\eta)} p(\xi,\eta)G_{ij}^{(B)}(\xi-x,\eta-y,z)d\xi d\eta+\\&\iint\limits_{S(\xi,\eta)} q(\xi,\eta)G_{ij}^{(C)}(\xi-x,\eta-y,z)d\xi d\eta+\\&+\sigma_{ij}^{(M)}(x,y,z)+\sigma_{ij}^{(N)}(x,y,z)+\sigma_{ij}^{(Q)}(x,y,z). \end{aligned} \quad (12)$$

Indeed, from the viewpoint of Tribo-Fatigue, the analysis of Eqs. (6) and (7) is possible from the following two positions. On the one hand, it is interesting how the field of the stresses caused by volumetric deformation is excited in a local area in which the field of contact stresses is simultaneously excited. Such an analysis is useful when the *direct effect* occurs in a Tribo-Fatigue system [1, 24, 57, 75, 101]. In essence, these are *problems of theory of elasticity without the St. Venant's principle*; all of them should make up a new domain of this theory [1, 25, 62]. On the other hand, it is interesting how the local field of the contact stresses varies under the effect of the superimposed field of the stresses caused by volumetric deformation. Such an analysis is useful when the *back effect* occurs in the Tribo-Fatigue system [57, 75, 101]. In essence, this is a *new class of problems of contact mechanics* [102, 103]. It is clear that Eqs. (6) and (7) allow one to make such analysis both quantitatively and qualitatively under nearly any loading conditions.

Some results of numerical calculations according to equation (12) as applied to the *roller/shaft (when bending)* Tribo-Fatigue system are presented in Figure10. It is seen how radically the combined stress-strain state (see Figure 10, *c*, *d*) differs from the stress state of the shaft at its console bending (see Figure 10, *b*) or from the field of the contact stresses without bending (see Figure 10, *a*). Clearly, the character of the stress-strain state of the shaft being bent in the zone of its contact with the roller differs considerably in the cases when contact occurs in the tension area (see Figure 10, *c*) and in the compression area (see Figure 10, *d*) during bending. It is clear that, if these factors are ignored in practical calculations of real systems, the estimations of the efficiency and durability of the latter will be wrong.

Clearly the fields of contact stresses in the corresponding sections are the same for both members of a friction pair in contact. The situation is, however, not the same in a Tribo-Fatigue system: pure contact stresses occur only in one of its members, i.e. in the member which does not undergo volumetric deformation. For example, when analyzing the *roller-shaft* system, combined stress state (12) appears in the working zone of the shaft, i.e., the main element of the Tribo-Fatigue system. This element experiences contact and volumetric deformation simultaneously. Here, the stress state is caused by two types of loads: the surface load (contact) and the volumetric load (see Figure 10, *c*, *d*). Naturally, it coincides with neither the pure contact stress state (in the working zone of the friction pair) nor the volumetric stress state (in the working zone of the structure element).

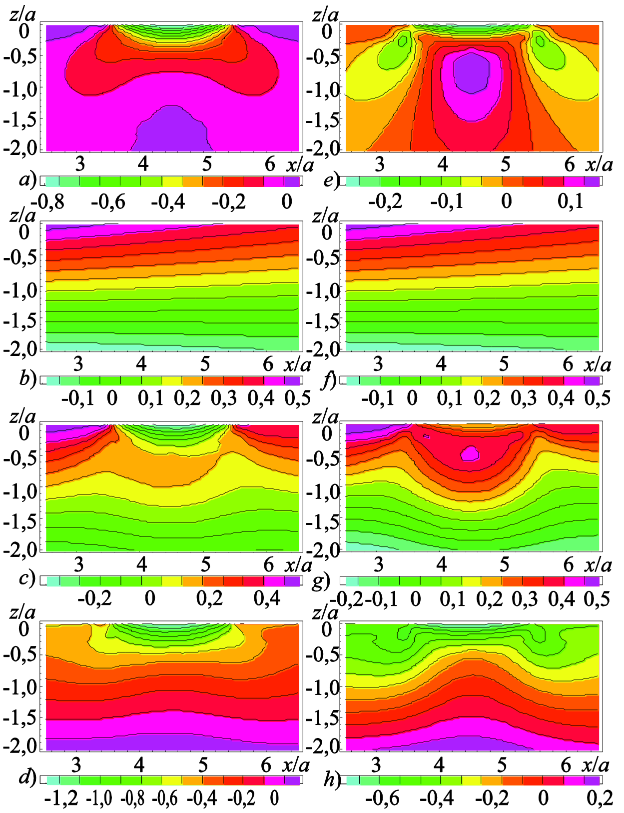

Figure 10 – *Roller / shaft* system: distributions of stresses $\sigma_{xx}^{(n)}$ (*a*), $\sigma_{xx}^{(b)}$ (*b*), $\sigma_{xx}^{(n)} + \sigma_{xx}^{(b)}$ (*c*), and $\sigma_{xx}^{(n)} - \sigma_{xx}^{(b)}$ (*d*) divided by p_0 and deformations $\varepsilon_{xx}^{(n)}$ (*e*), $\varepsilon_{xx}^{(b)}$ (*f*), $\varepsilon_{xx}^{(n)} + \varepsilon_{xx}^{(b)}$ (*g*), and $\varepsilon_{xx}^{(n)} - \varepsilon_{xx}^{(b)}$ (*h*) divided by p_0 / E (*E* is modulus of elasticity) in vicinity of contact on the plane $y = 0$ at $a / b = 0.5$

An analysis of the stress-strain state presented in Figure 10 is carried out for one component only (σ_{xx}, ε_{xx}). Figures 11–13 present the similar analysis of another Tribo-Fatigue system – the *roller / ring* [1, 24, 100] one – but it is carried out for all three stress components: σ_{xx} (Figure 11), σ_{yy} (Figure 12), and σ_{xy} (Figure 13). These calculations are performed for various ring thickness *h*, which changes within the range from 2.0 to 10 mm; the latter case (*h* = 10 mm) is almost the same as the *roller / roller* system (the contact pair). The Figures show that the stress distributions in the contact pair (at *h* = 10 mm) are practically symmetrical relatively to the *x*-axis, while the distribution patterns and the values of the stresses for the roller and ring in the Tribo-Fatigue system differ considerably. The transition from the contact system to the Tribo-Fatigue one becomes noticeable at a ring thickness of 7.5 mm. It is clearly seen that the stress state of the roller is caused by contact interaction only, while that of the ring in the vicinity of the initial contact point is "transient" and is caused by the bending of the ring as well.

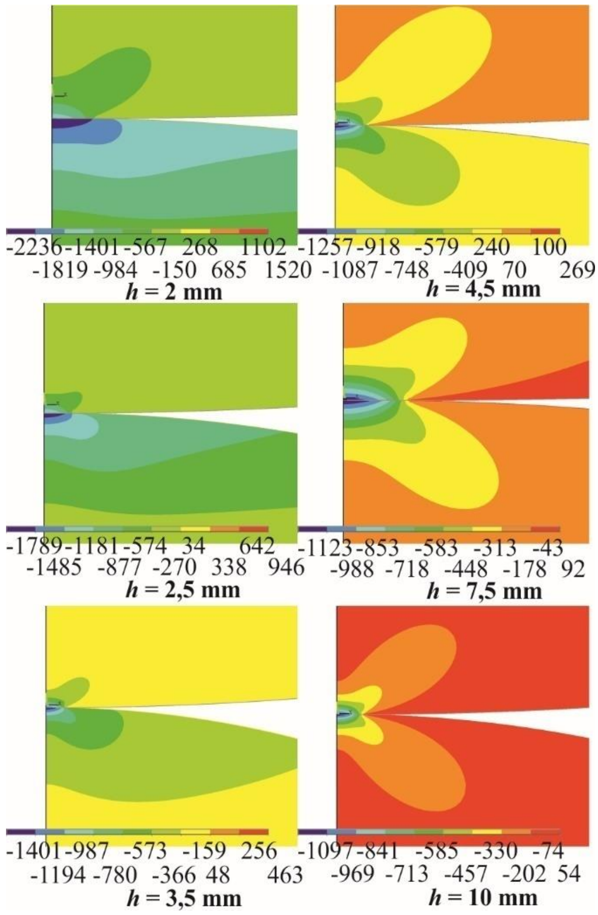

Figure 11 – *Roller / ring* system: distribution of stresses σ_{xx} (MPa)

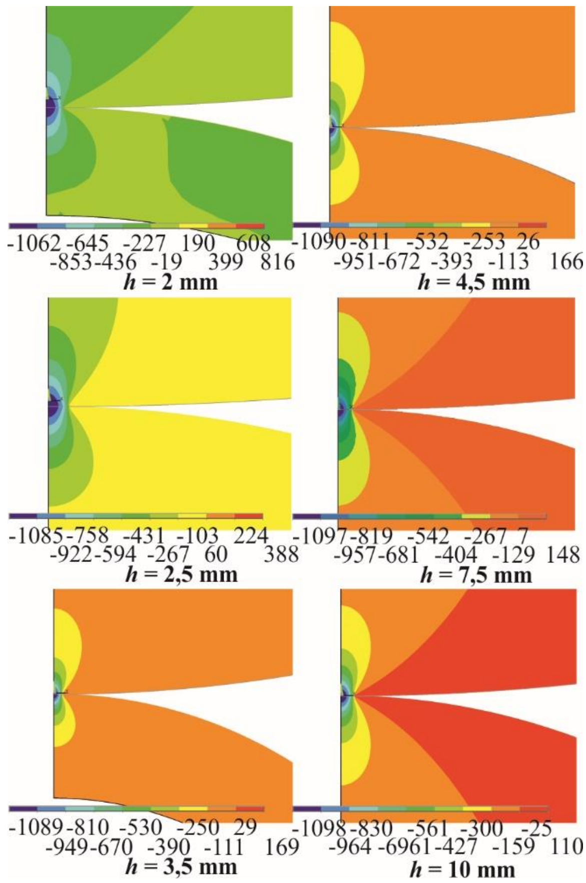

Figure 12 – *Roller / ring* system: distribution of stresses σ_{yy} (MPa)

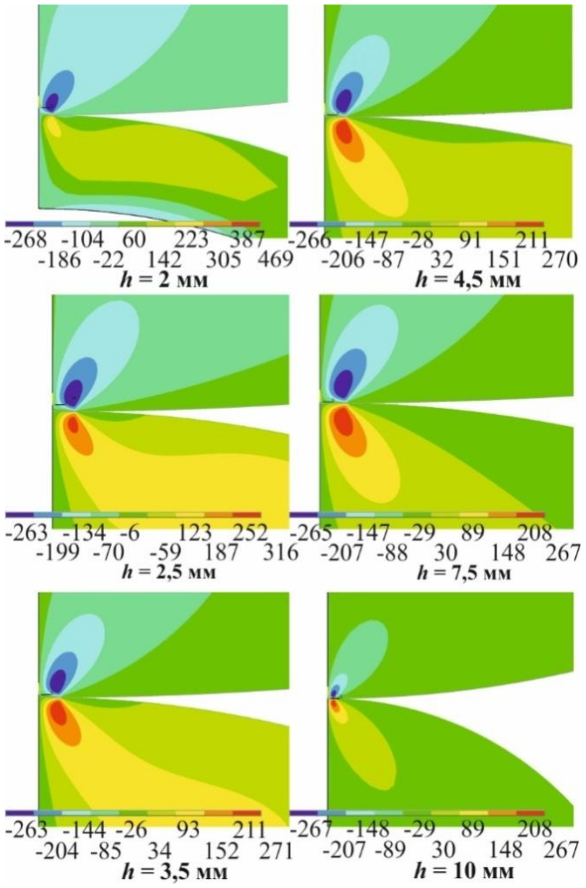

Figure 13 – *Roller / ring* system: distribution of stresses σ_{xy} (MPa)

Thus, the calculation results demonstrate eth substantial effect of ring bending on the stress state in the contact zone owing to the action of the united force F_N applied to the system. A similar statement is also true for the stresses and displacements.

Problem formulation and the substantiation of model (12) including photoelasticity experiments as well as the results of some studies of the stress state of Tribo-Fatigue systems are reported in works [1, 24, 63, 104–112].

Damage State. Naturally, the next level of complexity arises when introducing a measure of damage, which would be suitable for the quantitative assessment of the corresponding state of not only the material but also the system as a whole. Let us consider this problem in relation to the analysis of damage to specific objects.

Structure Element. A great number of works are devoted to the study of the fatigue resistance of specimens of various volumes determined by a restricted zone of finite dimensions with the critical stress level [113–142]. Complexities arise as to the substantiation of the criterion for restricting this zone and the criterion for assigning the damage level of the stresses in it. Weibull [124] treated this zone as the volume wherein the effective stress exceeds the lower strength limit of a defect element of the solid structure. However, this lower limit was considered to be zero and therefore the volume turned out to be simply the specimen volume. Kuguel [122] related the fatigue resistance to the "highly stressed" specimen volume, i.e., its domain, wherein the stresses exceed the arbitrarily assigned 95% threshold of the maximal stress in the dangerous section. The error associated with this assumption turned out to be great. The authors of references [113–117] substantiated the quantitative solution of the problem on the determination of the restricted domain of finite dimensions with the critical level of normal stresses in the specimen. This domain is called *dangerous volume*. As pointed out by Bolotin [127], any solid is a statistical ensemble of a great number of the initial elements each being responsible for the strength of the solid as a whole to some extent. To determine the "measure of the responsibility" of each elementary volume for the strength of the deformable solid as a whole, the notion of the dangerous volume of the solid must be introduced. If the deformable solid is characterized by complex stress state involving fatigue fracture, then in the general case the solid consists of both *volumes with the safe and dangerous states*.

The border between the volumes of dangerous and safe states is generally diffuse and probabilistic in nature. Dangerous volume $V_{P\gamma}$ increases with the increase of the probability P of failure of a body. But for the given value of P it may quantitatively vary depending on confidence probability level γ_0. Hence for P= const

$$V_{P_{\gamma\,\min}} \le V_{P_\gamma} \le V_{P_{\gamma\,\max}} \tag{13}$$

if $\gamma_{\min} \le \gamma_0 \le \gamma_{\max}$ where $\gamma_{\min}$, $\gamma_{\max}$ form the interval of permissible change. Assuming that γ_0 = const then a dangerous volume will have a unique value associated with the probability of failure P.

In problems associated with wear, there exists a similar region that is often referred to as *the severely deformed* region (SDR). These have been determined experimentally. Physically, it makes good sense that wear occurs near the surface and in the SDR. The determination of the thickness of SDR, while important, is not an easy task, at least theoretically. This is an interesting area of research [143, 144].

According the *statistical model of a deformable solid* developed by Sosnovskiy, the dangerous volume is the volume V_P, wherein the occurrence of the cyclic stresses σ exceeding the lower limit $\sigma_{-1\min}$ of fatigue limit σ distribution is possible with a probability P (Figure 14). Below we shall briefly describe the model.

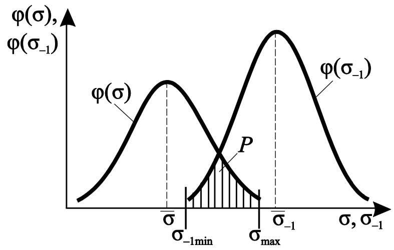

Figure 14 – Densities of distributions of probabilities of effective (σ) and limiting (σ_{-1}) stresses

If the distribution of the stresses σ in the three-dimensional space and the value of the criterion $\sigma_{-1\min}$ are known, then in the simplest case of the inequality to zero of only one component of the stress tensor the dangerous volume is calculated by the following formula

$$V_{P\gamma} = \iiint\limits_{\sigma(x,y,z)\ge\sigma_{-1\min}} dxdydz \,. \tag{14}$$

From (14) the *generalized condition of fatigue fracture* (the failure condition or the condition of achieving the limiting state) follows

$$V_{P\gamma} > 0 \tag{15}$$

with a probability P at the confidence probability γ.

If

$$V_{P\gamma} = 0, \tag{16}$$

then fatigue fracture cannot occur physically since in this case $\sigma < \sigma_{-1\min}$ in any elementary volume. Relation (16) is the *generalized condition of nonfracture* (the condition of safe operation).

Let us now consider the formation of dangerous volumes in the simplest structure elements such as cylindrical and prismatic specimens during bending. In this case the stress state is uniaxial and the distribution of the normal stresses over the section is described by the linear dependence. Then integral (14) can be calculated proceeding from simple geometrical plotting using the following formula [114]

$$\frac{V_{P\gamma}}{V_k} = \lambda\left(1-\frac{\sigma_{-1\min}}{\sigma}\right)^{\beta}\left(1+\frac{\sigma_{-1\min}}{\sigma}\right)^{\alpha_1}\times \\ \times\left(2+\frac{\sigma_{-1\min}}{\sigma}\right)^{\alpha_2} = \omega_\sigma, \tag{17}$$

where V_k – the working (total) volume of the deformable solid;

β, α_1, α_2 – known coefficients.

The ratio of the volumes $V_{P\gamma} / V_k = \omega_\sigma$ is the measure of the damage of the solid.

Thus, the dangerous volume is the domain of a cyclically deformed solid, wherein fatigue damage nucleates and initial fatigue cracks propagate. Probability of the formation of fatigue cracks is different at each "point" of the dangerous volume – it is greater at the points where the effective stresses are higher. Referring to Figure 15, *b*, one can easily find that the first cracks will appear most probably in the vicinity of point *A*. As the distance from *A* to the equal stress plane *BA′* (here, $\sigma = \sigma_{-1\min}$ = const) decreases the probability of their appearance reduces to the minimum and the cracks nucleate more probably on the surface *AA′* rather than in the subsurface layer *AB*.

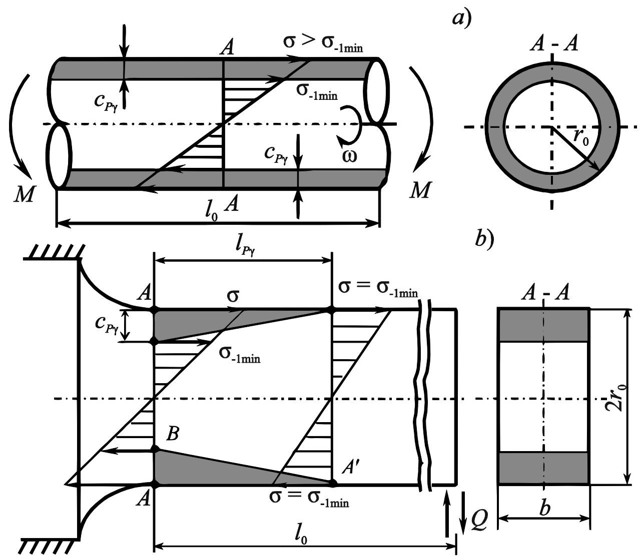

Figure 15 – Diagram of formation of dangerous volume during pure bending of shaft (*a*) and cantilevered bending of plate (*b*)

This is caused by the fact that the state of the surface is energetically specific (surface atoms have bonds only from the base part of the metal and therefore they are not balanced and have an excessive energy; that is why they tend to react with the environment more actively). Nevertheless, subsurface fracture is also possible since the strength characteristics in the bulk of the metal are somewhat poorer than near the surface. Special observations show that this forecast corresponds to the real pattern of the scattered damage of fine fatigue cracks [114]. Discussion of relevant experimental results is given for example in [23].

Friction Pair. Similar to the case of mechanical fatigue, the dangerous volume during contact fatigue is the volume V_P, wherein the normal contact stresses exceeding the lower limit $p_{f\min}$ of contact fatigue limit distribution $p_f = \sigma_{zz}^{(\max)}$ are possible to occur with a probability *P*. Here $\sigma_{zz}^{(\max)}$ is the pressure in the center of contact under the limiting contact load. The lower limit of the scatter of the fatigue limits for the tangential stresses is determined in a similar way. The theory of *volume damage when solving the contact problem* was developed by Sherbakov, Zhuravkov, and Sosnovskiy [24, 25, 128, 145–154]; below we present its brief description.

Let us consider the probability of the local damage at a point due to the effect of σ_{ij} (Figure 16).

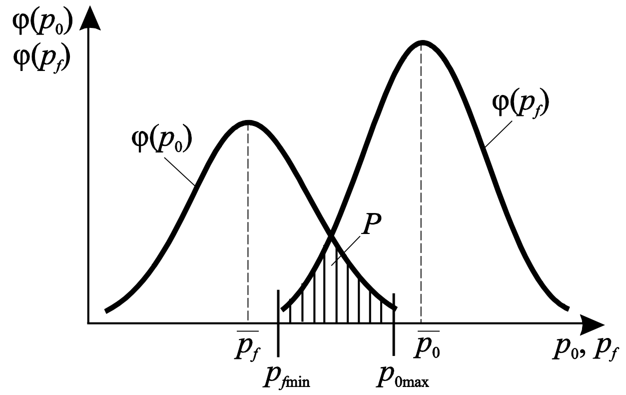

Figure 16 – Densities of distribution of probabilities of effective (p_0) and limiting (p_f) stresses

$$P(\xi \le 1) = \int_{-\infty}^{1} \phi(\xi)\, d\xi, \tag{18}$$

where $\xi = \sigma_{ij} / p_{f\min}$.

If the distribution of the stresses σ_{ij} in the three-dimensional vicinity of the contact zone and the value of the criterion $p_{f\min}$ are known, then in the case of the nonzero components of the stress tensor the dangerous volumes can be calculated *for each component*:

$$V_{P\gamma ij} = \iiint\limits_{\sigma_{ij}(x,y,z) \ge p_{f\min}} dxdydz \tag{19}$$

or by the stress intensity

$$V_{P\gamma \mathrm{int}} = \iiint\limits_{\sigma_{\mathrm{int}}(x,y,z) \ge p_{f\min}} dxdydz. \tag{20}$$

Table 3 summarizes the formulas for calculating of dangerous volumes and corresponding measures of damage for cases of the three-dimensional stress-strain state.

Table 3 – **Classification of static dangerous volumes (in case three-dimensional stress-strain state) [24]**

Dangerous volume type	Definition	Calculation formula	Relative measure of damage
Component	$V_{ij}=\{dV/\lvert\psi_{ij}\rvert\ge 1, dV\subset V_k\}$ The domain of a loaded solid, at each point of which the value of the corresponding component of the stress tensor greater than the limiting value	$V_{ij}=\iiint_{\lvert\psi_{ij}(V)\rvert\ge 1} dV$	$\omega_{ij}=V_{ij}/V_k$
Principal	$V_i=\{dV/\lvert\psi_i\rvert\ge 1, dV\subset V_k\}$ The domain of a loaded solid, at each point of which the value of the corresponding principal stress greater than the limiting value	$V_i=\iiint_{\lvert\psi_i(V)\rvert\ge 1} dV$	$\omega_i=V_i/V_k$
Spherical	$V_S=\{dV/\lvert\psi_S\rvert\ge 1, dV\subset V_k\}$ The domain of a loaded solid, at each point of which the value of the component of the spherical part of the stress tensor greater than the limiting value	$V_S=\iiint_{\lvert\psi_S(V)\rvert\ge 1} dV$	$\omega_S=V_S/V_k$
Deviator	$V_D=\left\{dV/\max\limits_{i,j}\lvert\psi_{ij}^D\rvert\ge 1, dV\subset V_k\right\}$ The domain of a loaded solid, at each point of which the value of at least one component of the deviator part of the stress tensor greater than the limiting value	$V_D=\iiint_{\max\limits_{i,j}\lvert\psi_{ij}^D(V)\rvert\ge 1\ge 1} dV$	$\omega_D=V_D/V_k$
Combined	$V_C=\bigcap\limits_{i=p,j=m}^{q,n} V_{ij}, i, j, p, q, m, n = x, y, z,$ $V_C=\bigcap\limits_{i=p}^{q} V_i, i, p, q = 1, 2, 3$ The intersection of two or more dangerous volumes	$V_C=\iiint_{\bigwedge\limits_{i=p,j=m}^{q,n}\lvert\psi_{ij}(V)\rvert\ge 1} dV$ $V_C=\iiint_{\bigwedge\limits_{i=p}^{q}\lvert\psi_{ij}(V)\rvert\ge 1} dV$	$\omega_C=V_C/V_k$
Octahedral	$V_{\text{int}}=\{dV/\psi_{\text{int}}\ge 1, dV\subset V_k\}$ The domain of a loaded solid, at each point of which the value of the stress intensity greater than the limiting value	$V_{\text{int}}=\iiint_{\psi_{\text{int}}(V)\ge 1} dV$	$\omega_{\text{int}}=V_{\text{int}}/V_k$
Tensor	$V_T=\bigcup\limits_{i=p,j=m}^{q,n} V_{ij}, i, j, p, q, m, n = x, y, z,$ $V_T=\bigcup\limits_{i=p}^{q} V_i, i, p, q = 1, 2, 3$ The integration of two or more dangerous volumes	$V_T=\iiint_{\bigvee\limits_{i=p,j=m}^{q,n}\lvert\psi_{ij}(V)\rvert\ge 1} dV$ $V_T=\iiint_{\bigvee\limits_{i=p}^{q}\lvert\psi_{ij}(V)\rvert\ge 1} dV$	$\omega_T=V_T/V_k$

Tribo-Fatigue System. In such system all six components of the stress tensor are nonzero. In the general case the limiting state (micro-crack formation) at a point of a tribosystem can be caused by several different components. Thus, the fatigue limit for the Tribo-Fatigue system is determined by each component of the stress tensor as the extreme value of its distribution under the effect of the limiting load.

In the general case of the effect of the limiting load $F_{*\lim}$ on the tribosystem we determine *the limiting values of each independent component of the stress tensor* $\sigma_{ij}^{(\pm *\lim)}$, *i*, *j* = *x*, *y*, *z*, each principal stress $\sigma_i^{(\pm *\lim)}$, *i* = 1, 2, 3, and the stress intensity $\sigma_{\text{int}}^{(*\lim)}$:

$$\begin{aligned}\sigma_{ij}^{(\pm *\lim)} &= \underset{dV}{\operatorname{extr}}\left[\sigma_{ij}\left(F_{*\lim}, dV\right)\right],\\ \sigma_{i}^{(\pm *\lim)} &= \underset{dV}{\operatorname{extr}}\left[\sigma_{i}\left(F_{*\lim}, dV\right)\right],\\ \sigma_{\text{int}}^{(*\lim)} &= \max_{dV}\left[\sigma_{\text{int}}\left(F_{*\lim}, dV\right)\right],\end{aligned} \quad (21)$$

where $\operatorname{extr}(y)=\begin{cases}\max(y), & \text{at } y\ge 0,\\ \min(y), & \text{at } y<0.\end{cases}$

Similarly, for a homogeneous isotropic deformable solid we determine *the limiting normal and tangential stresses* $\sigma_n^{(*\lim)}$ and $\sigma_\tau^{(*\lim)}$ as well as *the limiting principal stress* $\sigma_1^{(*\lim)}$ and *the limiting stress intensity* $\sigma_{\text{int}}^{(*\lim)}$:

$$\begin{gathered}\sigma_n^{(*\lim)} = \max_{dV,i}\left(\left|\sigma_{ii}\left(F_{*\lim}, dV\right)\right|\right),\ i=x,y,z;\\ \sigma_\tau^{(*\lim)} = \max_{dV,i,j}\left(\left|\sigma_{ij}\left(F_{*\lim}, dV\right)\right|\right);\\ i,j=x,y,z,\ i\ne j;\\ \sigma_1^{(*\lim)} = \max_{dV}\left(\left|\sigma_{1}\left(F_{*\lim}, dV\right)\right|\right)\end{gathered} \quad (22)$$

where *dV*– elementary volume of the loaded solid.

Therefore, if we consider the tensor σ_{ij} in each elementary volume *dV* of the solid, then in the general case we can introduce the following *three types of the tensor of the relative damaging stresses* to describe variations in the effective stresses compared to the limiting stresses, i.e., the component, principal, and octahedral:

$$\begin{gathered}\psi_{ij}=\sigma_{ij}/\sigma_m^{(*\lim)},\ m=\begin{cases}n & \text{if } i=j,\\ \tau & \text{if } i\ne j,\end{cases}\\ \psi_i=\sigma_i/\sigma_i^{(*\lim)},\\ \psi_{\text{int}}=\sigma_{\text{int}}/\sigma_{\text{int}}^{(*\lim)},\end{gathered} \quad (23)$$

where ψ_{ij}, ψ_i, ψ_{int}, generally speaking, are of probabilistic nature since under the strength conditions both the effective stresses σ_{ij}, σ_i, and σ_{int} and the limiting values $\sigma_k^{(*\lim)}$, $\sigma_i^{(*\lim)}$, and $\sigma_{\text{int}}^{(*\lim)}$ are random values with the corresponding distribution densities.

The tensor ψ_{ij} can be divided into the *spherical* ψ^S and the *deviator* ψ_{ij}^D parts:

$$\psi^S = \begin{pmatrix} \psi_s & 0 & 0 \\ 0 & \psi_s & 0 \\ 0 & 0 & \psi_s \end{pmatrix},$$

$$\left(\psi_{ij}^D\right) = \left(\psi_{ij}\right) - \psi^S = \tag{24}$$

$$= \begin{pmatrix} \psi_{11} - \psi_s & \psi_{12} & \psi_{13} \\ \psi_{21} & \psi_{22} - \psi_s & \psi_{23} \\ \psi_{31} & \psi_{32} & \psi_{33} - \psi_s \end{pmatrix},$$

where

$$\psi_s = \frac{\left(\sigma_{11} + \sigma_{22} + \sigma_{33}\right)}{3\sigma_k^{(*\lim)}} = \frac{1}{3}\left(\psi_{11} + \psi_{22} + \psi_{33}\right).$$

As applied to Tribo-Fatigue systems, the procedure of calculating dangerous volumes implies that, first, it is necessary to have the knowledge of the three-dimensional stress state of the elements caused by both local loading and volume deformation, and, second, the determination of the critical (limiting) stresses, which serve as the criterion for restricting the corresponding dangerous domains, is required.

Below we present foundations of the theory of the volume damage of Tribo-Fatigue systems developed by Sosnovskiy, Sherbakov, and Zhuravkov [24, 25, 128, 145–154].

The conditions for restricting the dangerous volumes are as:

$$V_{ij} = \left\{ dV \,/\, \left|\sigma_{ij}\right| \geq \sigma_m^{(*\lim)}, dV \subset V_k \right\}, \; i, j = x, y, z, \tag{25}$$

$$V_i = \left\{ dV \,/\, \left|\sigma_i\right| \geq \sigma_i^{(*\lim)}, dV \subset V_k \right\}, \; i = 1, 2, 3, \tag{26}$$

$$V_{\text{int}} = \left\{ dV \,/\, \sigma_{\text{int}} \geq \sigma_1^{(*\lim)}, dV \subset V_k \right\}. \tag{27}$$

Corresponding relative measures of damage of a body or a system are

$$\begin{gathered} \omega_{ij} = V_{ij} / V_k, \; \omega_i = V_i / V_k, \; \omega_{\text{int}} = V_{\text{int}} / V_k, \\ 0 \leq \omega_{ij} \leq 1, \; 0 \leq \omega_i \leq 1, \; 0 \leq \omega_{\text{int}} \leq 1. \end{gathered} \tag{28}$$

Since for a Tribo-Fatigue system the process of its damage during contact fatigue can be studied, then the working volume V_k in expressions (25)–(28) is not the whole volume of the deformable solid but dangerous volume determined by the value only the maximum permissible wear (Figure17).

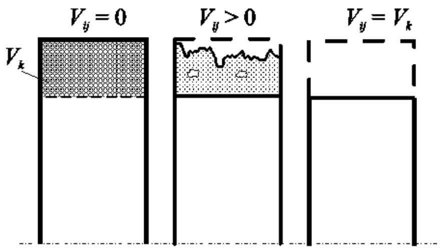

Figure 17 – Diagram of working and dangerous volumes during contact fatigue

To describe the pattern and extent of damage in the contact zone we introduce the notion of *types of dangerous volumes*. Tables 3 and 4 contain the definitions, designations, and formulae for calculating dangerous volumes and the summary of the corresponding (relative) measures of damage [24]. In Table 4 S are the projections of the corresponding dangerous volumes on the plane orthogonal to the trajectory of motion (l).

Table 4 – **Classification of dynamic dangerous volumes [24]**

Dangerous volume type	Definition	Calculation formula	Relative measure of damage
Component	$V_{ij}^d = \bigcup_l V_{ij}(t_l)$ The domain of a loaded solid, at each point of which the value of the corresponding component of the stress tensor greater than the limiting value at least once (at the moment of time t_l) during the loading cycle	$V_{ij}^d = \int_l S_{ij}(l)\,dl$	$\omega_{ij}^d = V_{ij}^d / V_k$
Principal	$V_i^d = \bigcup_l V_i(t_l)$ The domain of a loaded solid, at each point of which the value of the corresponding principal stress exceeds the limiting value at least once (at the moment of time t_l) during the loading cycle	$V_i^d = \int_l S_i(l)\,dl$	$\omega_i^d = V_i^d / V_k$
Spherical	$V_S^d = \bigcup_l V_S(t_l)$ The domain of a loaded solid, at each point of which the value of the component of the spherical part of the stress tensor is greater the limiting value at least once (at the moment of time t_l) during the loading cycle	$V_S^d = \int_l S_S(l)\,dl$	$\omega_S^d = V_S^d / V_k$
Deviator	$V_D^d = \bigcup_l V_D(t_l)$ The domain of a loaded solid, at each point of which the value of at least one component of the deviator part of the stress tensor greater than the limiting value at least once (at the moment of time t_l) during the loading cycle	$V_D^d = \int_l S_D(l)\,dl$	$\omega_D^d = V_D^d / V_k$
Combined	$V_C^d = \bigcap_{i=p,\, j=m}^{q,n} V_{ij}^d$,$i, j, p, q, m, n = x, y, z$, $V_C^d = \bigcap_{i=p}^{q} V_i^d$, $i, p, q = 1, 2, 3$ The intersection of two or more dynamic dangerous volumes	$V_C^d = \int_l S_C(l)\,dl$	$\omega_C^d = V_C^d / V_k$

Continuation Table 4

Octahedral	$V_{\text{int}}^{d}=\bigcup_{l} V_{\text{int}}(t_l)$ The domain of a loaded solid, at each point of which the value of the stress intensity is greater than the limiting value at least once (at the moment of time t_l) during the loading cycle	$V_{\text{int}}=\iiint\limits_{\psi_{\text{int}}(V)\geq 1} dV$	$V_{\text{int}}^{d}=\int_{l} S_{\text{int}}(l)\,dl$
Tensor	$V_T^d=\bigcup_{i=p,\,j=m}^{q,n} V_{ij}^d$, *i, j, p, q, m, n* = *x, y, z*, $V_T^d=\bigcup_{i=p}^{q} V_{ij}^d$, *i, p, q* = 1, 2, 3 The integration of two or more dynamic dangerous volumes	$V_T^d=\int_{l} S_T(l)\,dl$	$\omega_T^d=V_T^d/V_k$

Figure 18 shows the graphical interpretation of the component dangerous volumes V_{zz}, V_{yy}, and V_{xx}, their intersections (the combined dangerous volumes) $V_{xx}\cap V_{yy}$, $V_{yy}\cap V_{zz}$, $V_{xx}\cap V_{zz}$, and $V_{xx}\cap V_{yy}\cap V_{zz}$ as well as their integration $V_{xx}\cup V_{yy}\cup V_{zz}$ (the tensor dangerous volume).The Figure also illustrates the tensor $\psi_{ij}(dV)$ of the relative damaging stresses in accordance with representation (23). It is seen that, if the components of the tensor $\psi_{ij}(dV)$ are equal, the most dangerous zone, wherein the nucleation of fracture is expected, is the intersection of three component volumes $V_{xx}\cap V_{yy}\cap V_{zz}$.

Since dangerous volumes can be of an arbitrary and complex shape, their analytical determination can be complicated; in these cases they are calculated using the Monte-Carlo method.

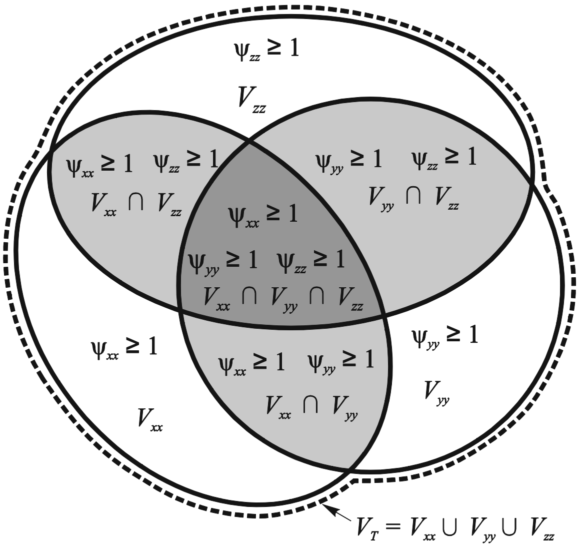

Figure 18 – Diagram of integration (dashed line) and intersections (shadowed area) of dangerous volumes due to effect of normal stresses

In the zones of three component volumes (V_{xx}, V_{yy}, V_{zz}) fracture is more likely to occur in the area where the tensor $\psi_{ij}(dV)$ has the maximal value. Generally, Figures like Figure 18 allow one to analyze various possible peculiarities of predicted damage. They appear, in the zones of the intersection (superposition) of the dangerous volumes resulted from either only the normal or only the tangential stresses, or the simultaneous effect of the normal and tangential stresses with different signs. Thus, the possibility exists to analyze specifically the role of the processes of tearing and shear in the formation of complex damage.

Let us consider simultaneously the dangerous volumes and the tensors of the damaging stresses at a point and introduce the *function of the damage of a unit volume*

$$d\Psi_q=\psi_q(V)\,dV\,, \tag{29}$$

where $q\in\{ij,i,\text{int}\}$, *ij*, *i*, int are the indices for component, principal and octahedral relative damaging stresses respectively.

Function (29), in essence, is the value of the specific damaging mechanical parameter.

Then the function of the damage of the volume *V* is

$$\Psi_q^{(V)}=\int_V \psi_q(V)\,dV\,. \tag{30}$$

We note that integration in (30) is carried out not over the total volume *V* but only over the tensor dangerous volume since, because of the definition of ψ_{ij}, $\psi_{ij}=0$ outside the dangerous volumes.

An example of calculating the volume of dangerous conditions in the contact interaction of the formulas given in the tables 3 and 4. We determine the dangerous volumes for the following initial data:

$$\begin{gathered}
p_0=\sigma_{zz}^{(n)}(F_c)\big|_{x=0,y=0,z=0}=2960\ \text{MPa},\\
p_{f\,\min}=p_0\left(F_c^{(*\lim)}\right)=888\ \text{MPa},\\
\sigma_n^{(*\lim)}=\max_{dV,i}\left(\left|\sigma_{ii}\left(F_c^{(*\lim)},dV\right)\right|\right)=\\
=p_{f\,\min}=0.3p_0,\\
\sigma_\tau^{(*\lim)}=\max_{dV,i,j}\left(\left|\sigma_{ij}\left(F_c^{(*\lim)},dV\right)\right|\right)=\\
=0.33p_{f\,\min}=0.09\,p_0,\\
f=0.5, b/a=0.5.
\end{gathered} \tag{31}$$

For the ratio of the contact ellipse axes $b/a=0.5$ equation (21) becomes

$$\sigma_{\text{int}}^{(*\lim)}=\max_{dV}\left[\sigma_{\text{int}}\left(F_{*\lim},dV\right)\right]=0{,}62\,p_{f\,\min}. \tag{32}$$

The results of the calculation of the component dangerous volumes for the limiting values from relations (31) are presented in Figures 19–22. The data in the Figures show that the dangerous volumes V_{zz}, V_{xz}, and V_{yz} corresponding to the maximal stresses $\sigma_{zz}^{(n)}$, $\sigma_{xz}^{(n)}$, and $\sigma_{yz}^{(n)}$ are maximal.

The analysis of Figures 21 and 22 allows us to draw the following two important conclusions. On the one hand, the *damage process in the general case is scattered*, i.e., initial cracks may nucleate at each point of the dangerous volume. On the other hand, the *damage process is discrete*, i.e., there are the local domains, wherein the appearance of initial cracks is most probable (e.g., in combined dangerous volumes, see Figure 22 or in the multiply connected domain of the tensor dangerous volume, see Figure 21). Proceeding from the data in Figures 21 and 22, we may carry out more profound analysis of damage as a result of the appearance, evolution, and interaction of dangerous volumes.

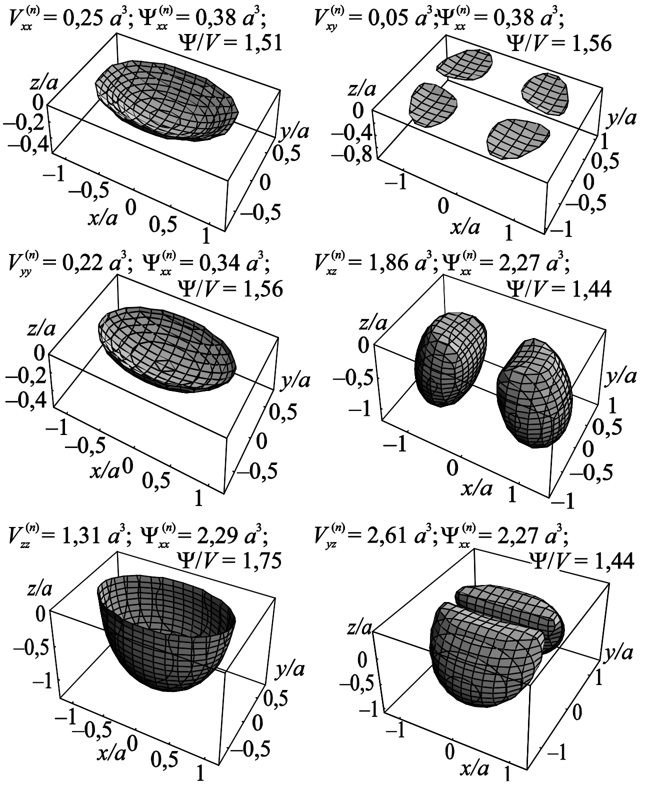

Figure 19 – Component dangerous volumes under effect of load $p(x, y)$, $b/a = 0.5$

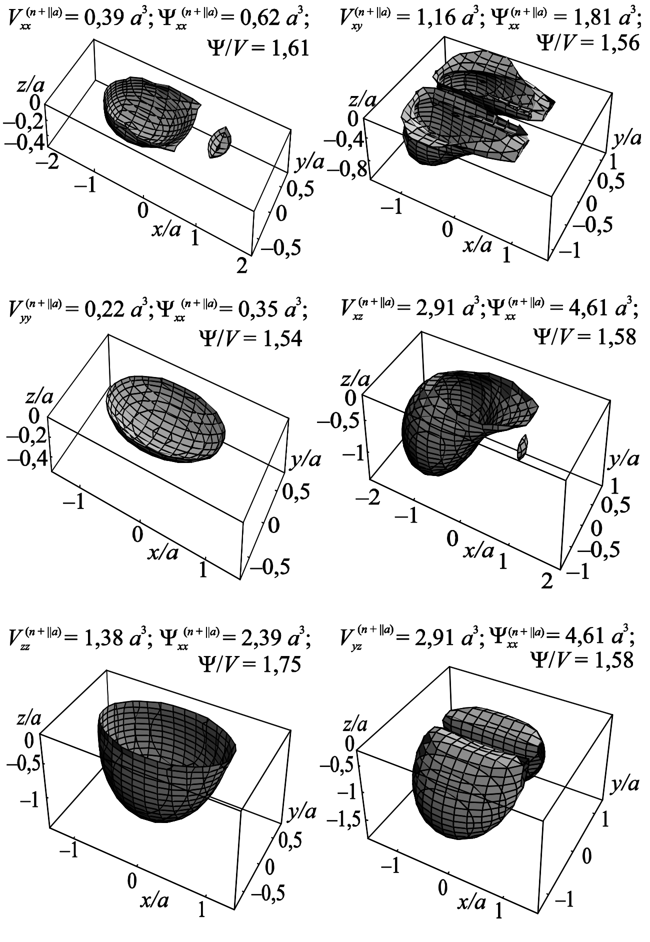

Figure 20 – Component dangerous volumes under effect of loads $p(x, y)$ and $q^{(\|a)}(x, y)$, $f = 0.5$, $b/a = 0.5$

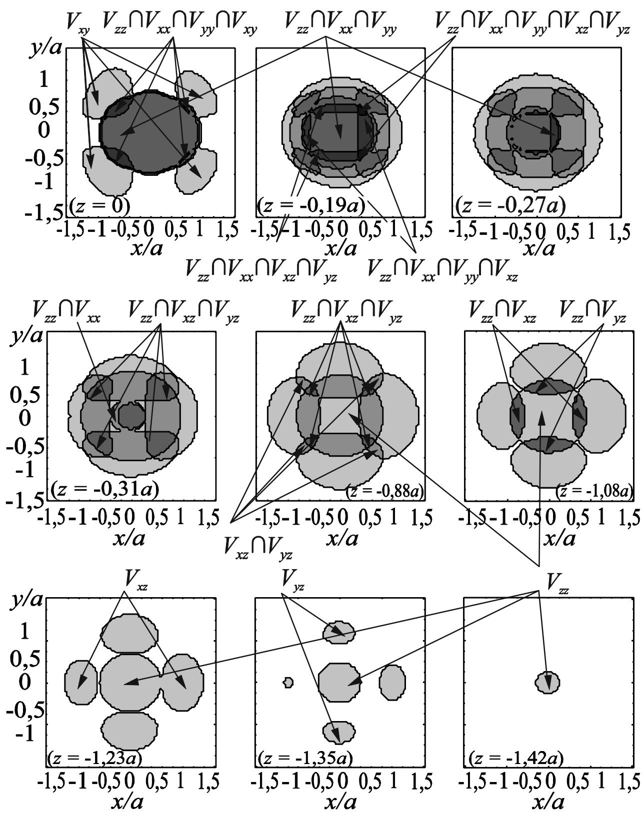

Figure 21 – Sections of dangerous volumes V_{ij} under effect of loads $p(x, y)$ and $q^{(\|a)}(x, y)$ along z-axis for $\sigma_n^{(*\lim)} = 0.3p_0$, $\sigma_\tau^{(*\lim)} = 0.09p_0$, $f = 0.05$, $b/a = 0.813$

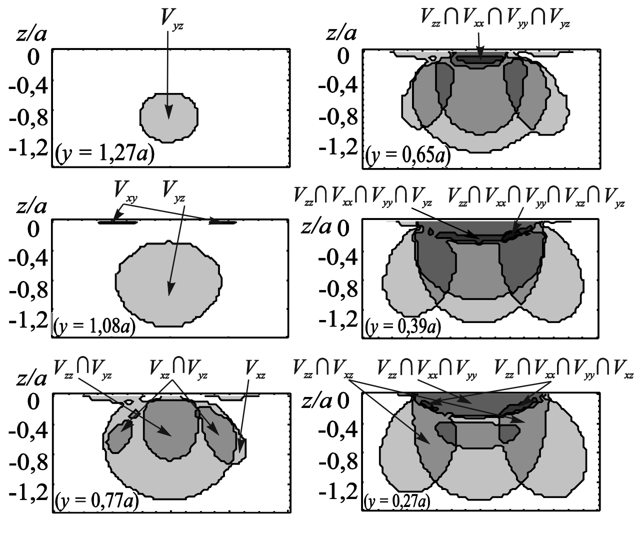

Figure 22 – Sections of dangerous volumes V_{ij} under effect of loads $p(x, y)$, $q^{(\|a)}(x, y)$ along y-axis for $\sigma_n^{(*\lim)} = 0.3p_0$, $\sigma_\tau^{(*\lim)} = 0.09p_0$, $f = 0.05$, $b/a = 0.813$

We note that the known *phenomenological theories of damage* (continuity) e.g., the Kachanov theory [130], the Rabotnov theory [131] etc. are based on a united interval of variation in the governing parameter

$$0 \le \omega \le 1,$$

similar to the condition (28). The summary and analysis of such approaches can be found in [132]. However, all these theories relate the parameter ω only to the dangerous section of a structure element and are concretized only by a certain *(sole) damage criterion*, while measures (25)–(27) and (28) are *volumetric by definition and multi-criteria*; they are called the Sosnovskiy – Sherbakov dangerous volumes and Sherbakov – Sosnovskiy damages, respectively.

Some results of the study of volumetric damage can be found in references [1, 24, 132, 148–151].

Limiting State. Clearly the state of a Tribo-Fatigue system is governed by different criteria. For example, the *fatigue fracture* (disintegration) of a structure element (e.g., a shaft) undergoing alternating (cyclic) loading can serve as a criterion for its limiting state and the *critical value of the wear* (during sliding) or the *critical density (depth) of pits* (during rolling) can serve as a criterion for the limiting state of a friction pair. Nevertheless, the failure of a Tribo-Fatigue system can occur by *all the above criteria of the limiting state* depending on the conditions of operation and testing.

It is found [155] that most of the known *theories* of reaching the *limiting state* (the so-called *strength theories*) of structure members are in line with the *hypothesis of Nadai* [156]

$$F(\tau_{\text{oct}}, \sigma_{\text{oct}}, m_i) = 0, \tag{33}$$

where τ_{oct}, σ_{oct} are the octahedral tangential and normal stresses; m_i are some parameters characterizing mechanical properties of the material.

A review of the available literature reveals that the problem of the development of specific theories of the limiting state (strength or wear resistance theories) has not been explicitly stated in Tribology. However, it is accepted that to assess the limiting state of a friction pair one can use one or another known strength theory. In fact, it is shown (see, e.g., [157]) that the *equivalent stress* is proportional to the specific friction force

$$\sigma_{eq} \sim fp_0 = \tau_W , \tag{34}$$

where p_0 is the maximal pressure on the contact site and f is the friction coefficient.

A more general problem of developing the theory of the limiting state for not just *a deformable solid or a friction pair but for a system*, which experiences both the contact and off-contact loads, is formulated and solved in Tribo-Fatigue. With account for (6), (7), and (33) such problem is formulated in the following general form [1, 24]:

$$\Phi(T^{(V,W)}, E^{(V,W)}, m_k, \Lambda_{i/j} ...) = 0 , \tag{35}$$

where m_k are some characteristics of the contacting materials; $\Lambda_{i/j}$ is the *function of interaction of irreversible damages caused by loads of different nature*. It is easy to see that formula (35) is more general than hypothesis of *Nadai* (33). In essence, the *Sosnovskiy's hypothesis* (35) gives origin to a *new period* in the development of strength theories whose specific feature is the *consideration of the dialectic interactions of irreversible damages* [1, 158–160]. The *principles* of such interactions are formulated as follows.

The irreversible damages caused by only the contact load (let their measure be ω_p) and the irreversible damages caused by only the cyclic stresses produced by off-contact loads (let their measure be ω_σ) interact in a complex manner ($\omega_\sigma \rightleftarrows \omega_p$) if they occur simultaneously and in one and the same area of the deformable solids. The result of their interaction

$$f_\Sigma(\omega_\sigma \rightleftarrows \omega_p) = \omega_\Sigma \tag{36}$$

is the measure (ω_Σ) of complex wear-fatigue damage. Then the simplest *hypothesis on interaction* is written as follows [1]:

$$f_\Sigma(\omega_\sigma \rightleftarrows \omega_p) = (\omega_\sigma + \omega_p)\Lambda_{\sigma/p} = \omega_\Sigma \, , \Lambda_{\sigma/p} \gtreqless 1, \tag{37}$$

where $\Lambda_{\sigma/p}$ is the function of interaction of irreversible damages caused by contact (p) and off-contact (σ) loads.

Problem (35) and *Sosnovskiy's principle of* Λ*-interactions* (37) are reported in [1, 51, 161, 162 and others]. Here we just point out their main features. According to (37), under certain conditions the development of wear-fatigue damage (ω_Σ) can be *accelerated* considerably if $\Lambda_{\sigma/p} >> 1$. This means that *spontaneous* loss of strength occurs predominantly in the system. On the contrary, under other conditions the development of wear-fatigue damage (ω_Σ) can be considerably *retarded* if $\Lambda_{\sigma/p} << 1$. This means that *spontaneous hardening* occurs predominantly in the system. Finally, a situation may arise when $\Lambda_{\sigma/p} = 1$, i.e. no interaction of the damages caused by different loads occurs. It is observed, for example, if the events of damage ω_σ and ω_p are antithetical, i.e. if they are excited in different zones (areas) of the loaded system.

Therefore, in general case complex wear-fatigue damage is describes as a result of the *complicated competition the processes of hardening and softening*); this result and its direction are characterized by the numerical value of the Λ-function: it may be greater, lesser or equal to the unity. This means that *Λ-interaction are of dialectic nature*.

Figure 23 presents the generalization of the above-stated taking into account the temperature of damage ω_T. *Irreversible damages* ω_{ij} *interact dialectically* (the interaction function $\Lambda_{ij} \gtrless 1$) *and form complex (general or integral) damage* ω_Σ (ω_{ij}, Λ_{ij}). As in (37) when $\Lambda_{ij} > 1$, *weakening* of the system occurs; on the contrary, if $\Lambda_{ij} < 1$, there is a *slowing of hardening* of system, and at $\Lambda_{ij} = 1$ *a stable ratio of the hardening-weakening mechanisms* takes place. It is postulated that hardening is also finite ($0 < \Lambda_{ij} <1$), while weakening can be "infinitely" great (for example, the decomposition of a material during nuclear explosion: $\Lambda_{ij} \to \infty$). Figure 23 illustrates an example of the analysis of the interaction of the damages (ω) due to the contact (ω_p), volumetric (ω_σ), and thermodynamical (ω_T) loads.

A more complex, comparatively with problem (35), problem of assessing the limiting state of a system operating in a corrosive medium (denoted by *Ch* - chemistry) under the action of the contact, alternating (off-contact) and heat loads is formulated and solved in Tribo-Fatigue [1, 24, 25, 75, 161, 162]. The most general energetic approach was used to solve such an intricate problem. This theory is based on a number of statements. One of them is that the limiting state is determined by not the total energy u supplied to the Tribo-Fatigue system but only its *effective* (*dangerous*) *part* $u^{eff} << u$ which is consumed for the generation and development of damages. The criterion of the limiting state is the condition when the effective energy u^{eff} reaches its critical value u_0 in some restricted area of the member of the Tribo-Fatigue system, i.e. in its *dangerous volume*. The energy u_0 is treated as the *fundamental constant* of the given substance; it is independent of test conditions, types of the supplied energy and damage mechanisms.

1 Complexity of approach

a) *Many things considered as a whole* $f(\omega_p, \omega_\sigma, \omega_T) = \omega_\Sigma$;

b) *Whole, considered as many things* $\omega_\Sigma = f(\omega_p, \omega_\sigma, \omega_T, \Lambda_{\sigma\backslash p}, \Lambda_{T\backslash M})$

2 Principle of Λ-interaction of irreversible damages

analysis

$$F_\Sigma\left[\left(\omega_p \rightleftarrows \omega_\sigma\right) \rightleftarrows \omega_T\right] = \left[\left(\omega_p + \omega_\sigma\right)\Lambda_{\sigma\backslash p} + \omega_T\right]\Lambda_{M\backslash T} = \omega_\Sigma \,, \quad \Lambda \gtreqless 1$$

synthesis

3 Organic unity of:

a) theory (fundamental mechanical-mathematical models of tribo-fatigue);

b) experiment (innovative tribo-fatigue tests);

c) practice (operational observations).

Figure 23 – Methodology of studying interaction of irreversible damages (in Tribo-Fatigue)

The generalized criterion of the limiting state of a Tribo-Fatigue system is written as follows [1, 24, 163–165]:

$$u_\Sigma^{eff}\left(T^{(V,W)}, E^{(V,W)}, T_\Sigma, Ch, \Lambda(V), m_k\right) = u_0 \,, \qquad (38)$$

where T and E are the stress and strain tensors, respectively; T_Σ is the temperature generated by all heat sources; m_k, k = 1, 2, … are some characteristics of properties of the contacting materials; $\Lambda_{i\backslash j}$ are the parameters (functions) of interaction of irreversible damages caused by the loads of different nature.

To select the effective parts from the total energy we introduce the coefficients $A_\sigma(V)$, $A_\tau(V)$ and $A_T(V)$ with the corresponding dimensions which determine the portion of the absorbed energy:

$$\begin{aligned} du_\Sigma^{eff} &= \Lambda_{M\backslash T}(V)\Big\{\Lambda_{\tau\backslash\sigma}(V)\Big[A_\sigma(V)T_\sigma \cdot\cdot dE + \\ &+ A_\tau(V)T_\tau \cdot\cdot dE\Big] + A_T(V)kdT_\Sigma\Big\} = \qquad (39) \\ &= \Lambda_{M\backslash T}(V)\Big\{\Lambda_{\tau\backslash\sigma}(V)\Big[du_\sigma^{eff} + du_\tau^{eff}\Big] + du_T^{eff}\Big\}, \end{aligned}$$

where T_τ is the tensor of friction-shear stresses (the shear tensor); T_σ is the tensor of normal stresses (tensile and compressive stresses) (the tear tensor); $\Lambda_{\tau\backslash\sigma}(V)$ and $\Lambda_{M\backslash T}(V)$ are the functions of interaction of the energies of different nature.

The fact that the coefficients A in general case may vary at different points of the volume V allows one to take into account the inhomogeneity of the medium.

With consideration of (39) criterion (38) becomes

$$\Lambda_{M\backslash T}(V)\Big\{\Lambda_{\tau\backslash\sigma}(V)\Big[du_\sigma^{eff} + du_\tau^{eff}\Big] + du_T^{eff}\Big\} = du_0 \,. \qquad (40)$$

In the particular case when $A_\sigma(V) = A_\sigma = \text{const}$, $A_\tau(V) = A_\tau = \text{const}$, $A_T(V) = A_T = \text{const}$, $\Lambda_{\tau\backslash\sigma}(V) = \Lambda_{\tau\backslash\sigma} = \text{const}$ and $\Lambda_{M\backslash T}(V) = \Lambda_{M\backslash T} = \text{const}$ the stress state is caused, first, by volumetric deformation for which one may neglect all components of the stress tensor except the component σ (one-dimensional tension and compression and plane pure bending) and, second, by surface friction for which one may neglect all components of the stress tensor except the component τ. Then (40) is reduced to the *energy criterion of Sosnovskiy – Makhutov – Bogdanovich* [1, 161, 162, 166, 167]

$$\Lambda_{M\backslash T}\left[\Lambda_{\tau\backslash\sigma}\left(A_\sigma\sigma^2 + A_\tau\tau^2\right) + A_T T_\Sigma\right] = u_0 \,, \qquad (41)$$

where

$$A_\sigma = \frac{a_\sigma}{1 - D_\sigma}, \; A_\tau = \frac{a_\tau}{1 - D_\tau}, \; A_\sigma = \frac{a_T}{1 - D_T}, \; 0 \le D_{\sigma,\tau,T} \le 1 \,.$$

Thus, the criteria in Eqs. (40) and (41) are just the solutions of problem (31) and equation (41) is the simplest particular case. The content of all the forms of the criterion is common: *the limiting state of a Tribo-Fatigue system occurs when the sum of the interacting effective components of the energy caused by force, heat, and frictional loading* (*with account for the processes of stress corrosion, thermal, and tribochemical corrosion*) *reaches its critical value, i.e. the energy of rupture of the interatomic bond* (*the initial activation energy of the fracture process*).

The accordance of criterion (41) with the results of numerous tests (over 300) with the accuracy less than 10% is found in works [1, 161, 162].

The graphical representations of equation (41) is shown in Figure 24 as Sosnovskiy – Makhutov – Bogdanovich multicriterial diagrams *1–5*, on which the lines correspond to the *typical types* of the limiting states of various Tribo-Fatigue systems. Here the ordinate axis serves as the strength scale and the abscissa axis serves as the tribological scale.

The usual fatigue tests (no friction occurs, so that $\tau_W = 0$) result in determination of the fatigue limit of the shaft σ_{-1} (see Figure 24). During wear-fatigue tests of the Tribo-Fatigue system its value changes due to the effect of the processes of friction and wear (designated by $\sigma_{-1\tau}$ in Figure 24, *a*). This change governs the basic regularities of the direct effect. They can be described by characteristic curves *1–5* (see Figure 24, *a*) depending on the type of a Tribo-Fatigue system and conditions of its operation (the magnitude of contact load, temperature, properties of the environment, etc.). Curves 1 and 2 are typical for mechano-rolling fatigue, curves 2, 3 and 4 – for mechano-sliding fatigue, curves 3, 4 and 5 – for fretting fatigue under various conditions of testing.

The usual tests of the friction pair (no cyclic stresses occur, i.e. $\sigma = 0$) result in determination of the limiting value of friction stresses τ_f that is also called the limit of sliding fatigue(or the limiting contact pressure p_f that corresponds to the value of τ_f, see Figure 24). During wear-fatigue tests of the Tribo-Fatigue system its value changes due to the effect of the level of cyclic stresses (designated by $\tau_{f\sigma}$ in Figure 24, *b*). This change governs the basic regularities of the back effect. They can be similarly described by characteristic curves *1–5* (see Figure 24, *b*) depending on the type of a Tribo-Fatigue system and conditions of its operation (the magnitude of cyclic load, the temperature, the properties of the environment, etc.). Here curves *1–5* have the same sense with curves *1–5* in Figure 24, *a*. In both cases the value $\Lambda_{\sigma/p} < 1$ corresponds to curves of type *1* while the value $\Lambda_{p/\sigma} > 1$ corresponds to curves of types *3–5*. The significant difference is that in case of the direct effect, as it is noted above, the limiting state of the system is reached based on the criteria of resistance to mechanical fatigue while in case of the back effect – based on the criteria of friction and wear.

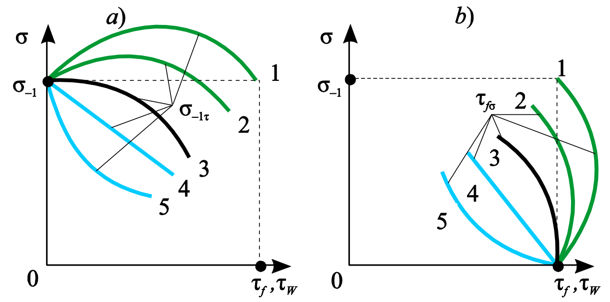

Figure 24 – Multicriteria diagrams of limiting states of Tribo-Fatigue systems: *a* – direct effect, *b* – back effect

The most general conclusions that can be made when analyzing equation (41) and Figure 24 are the following.

1 *The processes of friction and wear*, depending on the conditions of their occurrence, can both *reduce* significantly (see curves *3 – 5* in Figure 24, *a*) and *increase* much (see curves *1* and *2* in Figure 24, *a*) the *fatigue resistance* of a Tribo-Fatigue system. It means that *friction and wear in definite conditions of operation can be useful*. What is more, *by changing conditions of friction and wear* in a proper manner one can *control* effectively the processes of wear-fatigue damage of a specific Tribo-Fatigue system.

2 *Cyclic stresses*, depending on the conditions of testing, can both *reduce* significantly (see curves *3*, *4* and *5* in Figure 24, *b*) and *increase* much (see curves *1* and *2* in Figure 24, *b*) the *wear resistance* of the Tribo-Fatigue system. It means that *cyclic stresses are favorable in definite conditions of the system operation*. And more: *by changing the conditions of cyclic loading* in a proper manner, it is possible to *control* effectively the processes of wear-fatigue damage of a specific Tribo-Fatigue system.

Here we illustrate these conclusions by only two, most interesting, in our opinion, experimental facts.

The first example. Figure 25 [75, 168, 169] shows the results of testing of the *roller/shaft* Tribo-Fatigue system for mechano-rolling fatigue. The system is like those shown in Figure 4, *b* but rolling friction occurs only in the tension zone of the shaft being bent. The roller (counterspecimen) is made of alloyed steel 25ХГТ and the shaft (specimen) is made of carbon steel 45. The curves *ABCD* represent the diagram of the limiting states corresponding to curves *1* in Figure 24, *a*, *b*.

Within the portion AB the limiting state is reached due to the predominant development of a main fatigue crack while pit formation is the attendant process. Therefore, in this case the *direct effect* occurs. Oppositely, *within the portion CD* the limiting state is governed by the critical concentration of pits and the development of mechanical fatigue cracks is the attendant damage. Therefore, here we observe the *back effect*. The portion *BC* is a transition portion; within it the kinetic processes of interaction of the phenomena of friction (with wear) and mechanical fatigue run at high values of the loading parameters σ_a and p_0 which are close (or equal) to their critical values (σ_{-1} and p_f). Under these test conditions the limiting state can be reached following both criteria simultaneously.

The analysis of the diagram *ABCD* allows us to make the following basic conclusions.

(1) The fatigue limit of the specimen increases up to 1.5–1.6 times if rolling friction runs simultaneously (the direct effect – the portion *AB*).

(2) The critical (limiting) pressure in rolling friction increases up to 1.2–1.25 times if cyclic stresses are excited simultaneously in the specimen (the back effect – the portion *BC*).

(3) In the optimal region of contact stresses ($p_0 \approx 400 \ldots 1300$ MPa) wear during rolling leads to a significant rise of the system reliability by the fatigue resistance criterion, therefore, striving for wearless friction is undue.

(4) Under cyclic loading at optimal conditions ($\sigma_a \approx 50 \ldots 100$ MPa) tensile stresses are favorable since they promote increase in the system reliability by the rolling resistance criterion.

The results shown in Figure 25 represent one of the *surprises of Tribo-Fatigue*, namely, surprise S4 (*a gift from Tribo-Fatigue*) [170]. Indeed, this is a gift for a designer. The carrying capacity and durability of a Tribo-Fatigue system can be significantly increased without use of expensive materials or new technologies; it is necessary only to organize its operation (exploitation) under certain conditions.

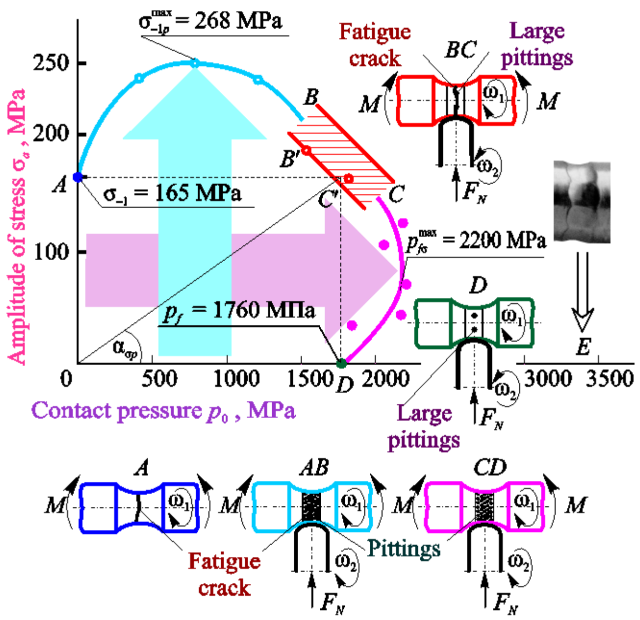

Figure 25 – Multicriterial diagram of limiting states of Tribo-Fatigue system at mechano-rolling fatigue

The improvement of the characteristics σ_{-1p} and $p_{f\sigma}$ of the limiting state during wear-fatigue tests compared to those in rolling (p_f) and mechanical fatigue (σ_{-1}) can be explained from the viewpoint of mechanics by the following main reasons [24, 75]:

– the addition of the stresses having opposite signs (the contact and bending stresses) that shifts the average cycle stress towards negative values, hence, reduces the maximal cycle stress;

– the hardening of the specimen working part by surface plastic deformation;

– the occurrence of favorable compressive stresses;

– the healing of primary fatigue cracks at elastoplastic deformation during rolling.

The application of precise experimental analysis methods allows one to study and understand other features of complex damage at mechano-rolling fatigue. As an example, Figure 26 shows the results of studies (by atomic force microscopy) of the processes of cracking of carbon steel 45 specimens in rolling friction and during wear-fatigue tests in response to the magnitudes of contact pressure p_0 and the amplitude of cyclic stresses σ_a [171, 172]. The Figures (scan sizes are ~35×35 μm^2) show the morphology of cracks typical for the corresponding testing conditions. The histogram shows how the critical depth h of the damaged layer depends on the magnitude of the cyclic stresses (while the contact pressure remains unchanged p_0 = 2130 MPa). These experimental data enable to deduce the following conclusions.

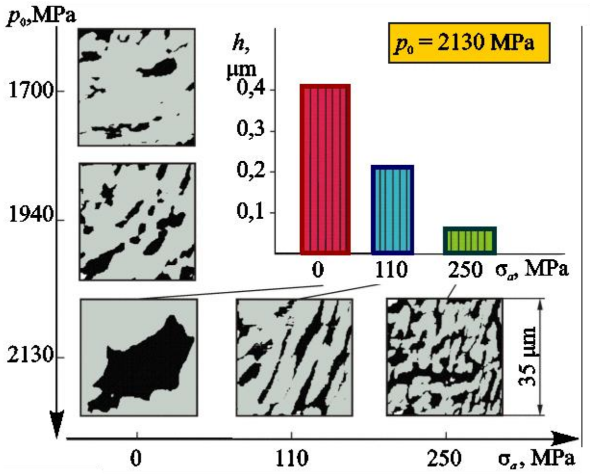

Figure 26 – Microtopography of surface damage in rolling friction (vertical column of Figures) and wear-fatigue tests (remaining Figures) [1, 15, 162]

In pure rolling friction any higher contact pressure intensifies plastic deformation, hence, it leads to deformation fragmentation of grains, appearance of initially *discrete pores and cracks* and then their chains. The system of deformed grains, chains of pores and cracks is unidirectional and oriented along the rolling direction. This process produces relatively large discrete pits. Two types of wear dominate, viz. *delamination and spalling*. The critical depth of the damaged layer is estimated ~0.4–0.5 μm.

During wear-fatigue tests deformation fragmentation of grains and appearance of pores and cracks are also observed. However, the pattern of damage changes significantly. As the amplitude of cyclic stresses increases, the process of formation of the second system of cracks accelerates, and now they cross the direction of rolling. Therefore, the damage becomes *scattered*, an almost equilibrium net of intersecting cracks and pores appears that fringe fine-dispersed particles (fragments of grains) of the material. The stronger the cyclic stress, the denser is the net of cracks and pores, the finer and thinner the separating particles; the critical depth of the damaged layer reduces to 0.05 μm. Thus, the appearance of large and deep pits is prevented, and they are not observed under these conditions. The prevailing process of wear is *surface crushing* in this case. It is characterized by separation of fine-dispersed particles from the working surface of the material that result from multiple microshearing over intersecting planes, formation of a huge number of scattered microscopic cracks and pores and fine crushing of grains. Such mechanism of complex surface damage is called the *scattered effect of multiple microshearing* (SEMMS) or the *effect of Sosnovskiy – Makhutov – Chizhik* [1, 15, 171].

The above results enable to establish additionally the following causes why wear-fatigue damage in definite conditions happens to be less dangerous than damage in friction (under similar contact pressures).

1 Superposition of the fields of contact and bending stresses leads to stronger dissipation of applied energy in a finer surface layer of the material and the localization of the processes of cracking and wear in it. The energy of deformation is consumed for finer crushing of fragments of grains and their multiple separation, rather than for penetration of damage deeper into the material.

2 Wear of the surface layer damaged by the net of cracks and pores exposes a fresh, relatively intact surface with a high fracture resistance. Thus, the appearance of relatively large pits, on the bottom of which a dangerous microconcentration of stresses and a dangerous main crack occur, takes more time or it is even prevented at all (depending on the loading conditions).

3 An approximately tenfold rejuvenation of the working surface by fragmentation, crushing and separation of metallic particles is required during wear-fatigue tests until the depth of damage is like that in rolling friction if the contact pressure coincides in both cases.

Thus, it is established experimentally that *wear-fatigue damage is a specific and peculiar type of surface damage* of the main element of a Tribo-Fatigue system. Its specific feature in these conditions is surface crushing resulting from the SEMMS running over intersecting planes of sliding. Its peculiarity is the following: notwithstanding the fact that it is a damaging process, it is useful because it leads to significant improvement of reliability and durability of a Tribo-Fatigue system. It is apparent that the optimum combination of the loading parameters σ_a and p_0 (see the wide arrows in Figure 25) creates such a state of a Tribo-Fatigue system when its load carrying capacity spontaneously and during prolonged time is maintained (or controlled automatically) by fine wear and removal of the damaged surface layer from the friction zone.

Here we should emphasize the difference in the terms wear-fatigue damage and fatigue wear. The term *fatigue wear* stresses the mechanism of wear during the repeated deformation of a thin surface layer in the area of contact between two bodies at sliding, rolling etc. The term *wear-fatigue damage* determines the result of the Λ-interaction of irreversible damages caused by the joint and simultaneous effect of both the contact load and volumetric forces of the cyclic deformation of at least one of the contacting system members (see also Table 2).

The second example. Here we describe briefly another surprise of Tribo-Fatigue (S3) called a *Tribo-Fatigue bomb* or the *Makhutov's surprise* [1, 170, 173]. The matter concerns the greatest breakdown, i.e. the damage of the rotor of a 1200 MW turbine by fretting fatigue cracks after four months of operation [173]. Its analysis is shown briefly in Figure 27.

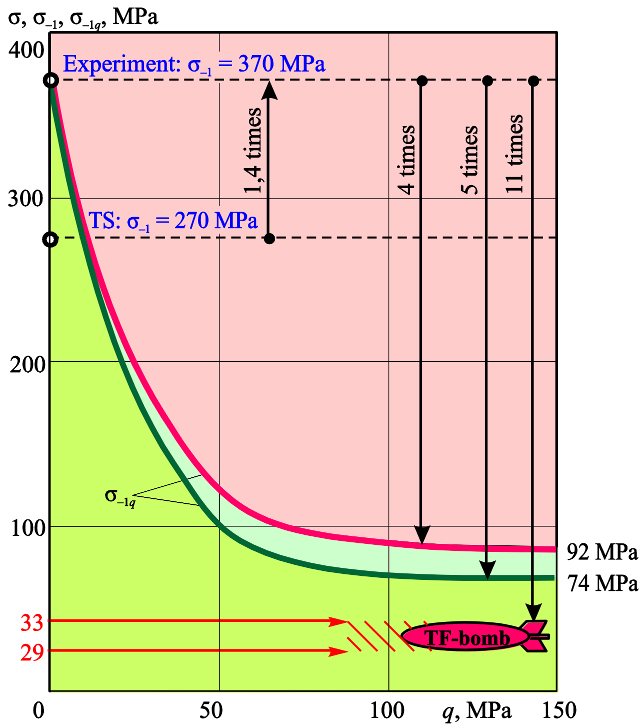

Figure 27 – To analysis of conditions of operational fail of rotor

According to the Technical Specifications (TS), the fatigue limit of the metal of the rotor should not be less than σ_{-1} = 270 MPa. In fact, from the data of experiments it was approximately 1.4 times higher (σ_{-1} = 370 MPa). It is well known (from the results of a huge number of tests carried out in different countries) that the combination of mechanical fatigue and fretting with broad variations of the contact pressure q leads to a significant drop of the fatigue limit σ_{-1q}: a 2…3-time drop is a common situation; a 4-time drop is a highly rare situation; a 5-time drop is the most pessimistic prediction that, as far as information goes, has not been confirmed experimentally. The *surprise S3* had been planted (Figure 27) seemingly in an obviously safe zone: the cyclic stresses $\sigma \approx$ 29–33 MPa were 11 times below the fatigue limit of the material of the rotor, the contact pressure < 150 MPa seemed just immaterial. Yet, the reality surpassed the worst expectations: the premature operational fail of the rotor occurred. The general conclusion is the following: the fatigue damage due to very small cyclic stresses is powerless by itself; but when it combines and *interacts* with (when it appears by itself) practically safe damage caused by contact stresses in definite (unfavorable) conditions acquires a formidable energy capable to put the most powerful rotor out of order. Thus, in this case the function of interaction of the mentioned damages is $\Lambda_{\sigma/q} >> 1$.

Papers [1, 68–72, 75, 165, 166, 173–188] represent a more detailed analysis of the limiting states of Tribo-Fatigue systems.

The diagram of the limiting states presented in Figure 27 is plotted by the criteria of the carrying capacity of systems. In a number of cases the wear (or the wear resistance) and durability characteristics turn out to be important. Therefore, test results can be presented as the diagrams of the limiting states plotted by the wear resistance and durability criteria.

Figure 28 shows the results of *mechano-sliding fatigue tests* of a Tribo-Fatigue system *steel 40Kh/polyamide Durethan BKV-30H*. Here the corresponding dependences are presented in the three-dimensional coordinates: the ultimate stress of the steel specimen – the contact pressure – the wear rate $I_\sigma(\sigma, p)$ of polymer counterspecimen; this is the *diagram of the limiting states by the wear resistance criterion* proposed first by L.A. Sosnovskiy [1]. According to Figure 28, the greater p_a and $I_\sigma(\sigma, p)$ the lower is the fatigue limit σ_{-1p} (spatial curve 1). Curve 4 characterizes the back effect (the influence of the wear rate on variations in σ_{-1p}), curve 2 describes the direct effect (the influence of the contact pressure on variations in σ_{-1p}), and curve 3 establishes the interrelation p_a – $I_\sigma(\sigma, p)$. Thus, the diagram contains almost full information on the resistance of the Tribo-Fatigue system to mechano-sliding fatigue by *the wear resistance criteria*.

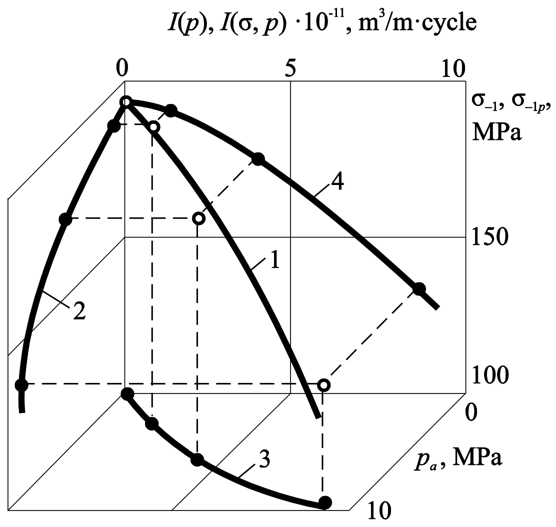

Figure 28 – Diagram of limiting states of metal-polymer Tribo-Fatigue system by wear resistance criteria

Figure 29 illustrates the results of *mechano-sliding fatigue tests* of the Tribo-Fatigue system *steel 40Kh/copolymer of formaldehyde SFD*. But in this case the corresponding dependences are presented in another coordinate system (compared to Figure 28), namely, the cyclic stresses σ – the wear rate $I_\sigma(\sigma, p)$ of the polymer counterbody – the cyclic durability $N(\sigma, p)$ of steel specimen; all tests were carried out under the constant contact pressure p_a = 5,7 MPa = const. Figure 29 is the *diagram of the limiting states by the durability criterion*.

According to Figure 29, the greater σ and $I_\sigma(\sigma, p)$ the lower is the durability $N(\sigma, p)$ (spatial curve 1). Curve 4 characterizes the back effect (the influence of the cyclic stresses on variations in $I_\sigma(\sigma, p)$), curve 3 describes the direct effect (the influence of the wear rate on $N(\sigma, p)$), and curve establishes the interrelation σ – $N(\sigma, p)$. Thus, this Sosnovskiy diagram [1] contains the full information on the resistance of the Tribo-Fatigue system to mechano-sliding fatigue by the durability or life criterion.

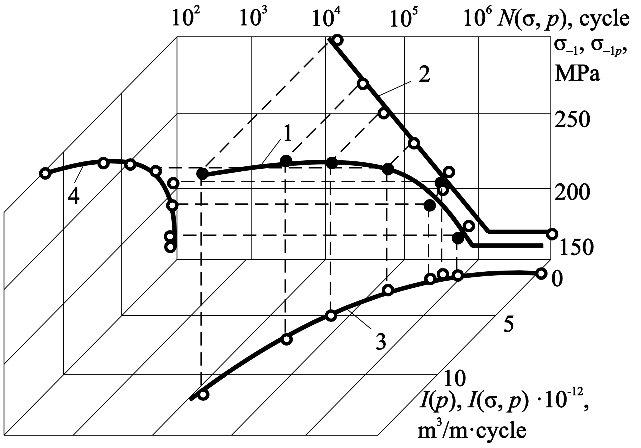

Figure 29 – Diagram of limiting states of metal-polymer Tribo-Fatigue system by durability criterion

Let us present some tests results [1, 188] for Tribo-Fatigue systems, according to which the damage processes resulted from contact interaction and the cyclic stresses ($\pm\sigma_a$) can intensify each other greatly. It is seen in Figure 30 that the cyclic stresses, depending on their sign, can either increase or reduce the wear i by 20–40 % and greater. The approach of the axes δ_c might be varied by cyclic stresses up to three times.

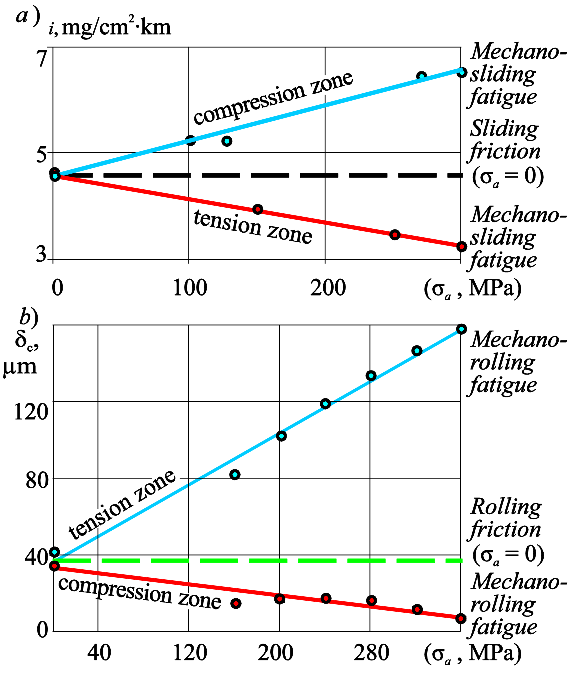

Figure 30 – Influence of cyclic stresses on sliding wear in system *steel (shaft) / cast iron (bearing insert)* (*a*) and on damage during rolling in system *rail steel (shaft) / steel 25 KhGT (roller)* (*b*)

The general regularities of the effect of the contact and bending (σ) stresses on *the wear rate* are described by the formula [1]

$$I_{p/\sigma} = K_0\left(\frac{fp_a}{\tau_f}\frac{\sigma_{-1}}{\sigma}\right)^{1+\beta_0 m_{\tau\sigma}} \cdot \frac{K_{overlap}}{N_{\tau\sigma}} K_{RR}, \qquad (42)$$

where the parameter of the asymmetry of wear-fatigue damage is

$$\frac{fp_a}{\tau_f}\cdot\frac{\sigma_{-1}}{\sigma} = \bar{\rho}, \qquad (43)$$

the Roscoe-Rehbinder parameter is

$$K_{RR} = \tau_f^{(surf)} / \tau_f^{film},$$

are the frictional fatigue limits determined with account of the influence of surfactants $(\tau_f^{(surf)})$ and oxide films (τ_f^{film}), $K_{overlap}$ is the overlapping factor, β_0 is the coefficient taking into account the influence of surface roughness, $m_{\sigma\tau}$ is the slope parameter of the frictional fatigue curve, K_0 is the empirical coefficient, $N_{\tau\sigma}$ is the durability by the wear criterion with account for the effect of the bending stresses.

Friction in Tribo-Fatigue System. In Tribology the friction force is believed to be depended only on one force factor, i.e. the normal contact load. For Tribo-Fatigue systems, in which the stress-strain state caused by both the contact and off-contact loads occurs in the united area, the stresses and strains due to the off-contact load form *additional boundary conditions on the contact surface*. This varies considerably the friction characteristics [1, 24, 52].

The friction force in the Tribo-Fatigue system F can be considered as a function of the common friction force at sliding (rolling) $\mathbf{F}^{(c)}$, which acts in the circumferential direction (the object is the friction pair), and the cyclic component $\mathbf{F}^{(b)}$ of the friction force, which appear additionally due to the excitation of off-contact (cyclic) stresses and strains. In general case the friction force in the Tribo-Fatigue system can be presented as the vector sum of the components $\mathbf{F}^{(c)}$ and $\mathbf{F}^{(b)}$ [189]:

$$\mathbf{F} = \mathbf{F}^{(c)} + \mathbf{F}^{(b)}, \qquad (44)$$

where $\mathbf{F}^{(c)} = f^{(c)}F_N$; $\mathbf{F}^{(b)} = f^{(c)}P_b$; $P_b = \int\limits_S p_b(s)ds$ is the additional contact load caused by the distribution of the contact pressure due to the action of the off-contact force

$$p_b = p_b\left(\sigma_{zz}\big|_{S(z=0)}, \mathbf{u}_b\big|_{S(z=0)}\right), \qquad (45)$$

where $z \perp S$; S is the contact area; $\sigma_{zz}\big|_{S(z=0)}, \mathbf{u}_b\big|_{S(z=0)}$ are the boundary conditions in the contact area due to the action of the off-contact force.

For the *Coulomb* dependence of the friction force for one of Tribo-Fatigue systems on the normal load it was found that [189, 190]

$$f = F/F_N = \left(F^{(c)} + F^{(b)}\right)/F_N = \left(f^{(c)}F_N + f^{(c)}P_b\right)/F_N =$$

$$= f^{(c)}\left(1 + P_b/F_N\right) = f^{(c)}\left[1 + k_{\sigma/p}\left(\frac{\sigma_{yy}^{(b)}}{p_0}\right)\right], \qquad (46)$$

where

$$k_{\sigma/p}\left(\frac{\sigma_{yy}^{(b)}}{p_0}\right) = \frac{F^{(b)}}{F_N} = \frac{3p_b}{2p_0} = -\frac{3\nu}{4}\frac{\sigma_{yy}^{(b)}\big|_{z=0}}{p_0}.$$

The analysis of formula *Sherbakov – Sosnovskiy* (46) and Figure 31 shows that if the contact and off-contact loads act simultaneously in the Tribo-Fatigue system, the total coefficient (force) of rolling friction in the tension zone decreases compared to the coefficient (force) of pure rolling friction, while in the compression zone it, in contrast, increases. Figure 32 illustrates the experimental confirmation of these regularities [62]. Here each point on the dependence $\bar{f}_\sigma - (\pm\sigma_a)$ is the average of 66 measurements. The experimental data were approximated by the following equation:

$$f_\sigma = f_r \pm a_r \frac{\sigma_a}{p_0}, \tag{47}$$

which is similar to (46). Here f_σ is the friction parameter in the *roller / shaft* Tribo-Fatigue system with account for the effect of cyclic stresses at bending (the subscript σ); f_r is the friction coefficient in the *roller / roller* pair (when $\sigma_a = 0$); a_r is the parameter. Table 5 presents the analysis of the experimental results, which shows that the error of formula (47) is below 7% (under the given test conditions).

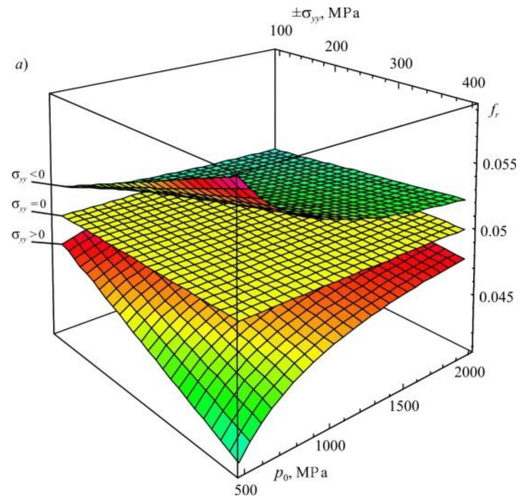

Figure 31 – Rolling friction coefficient depending on contact (p_0) and off-contact ($\sigma_{xx}^{(b)}$) loads for $k_{\sigma/p} = 0.225$, $\sigma_{xx}^{(b)}\Big|_{z=0}/p_0$ (tension zone $\sigma_{xx}^{(b)} > 0$ at the bottom, compression zone $\sigma_{xx}^{(b)} < 0$ at the top; plane between them corresponds to rolling friction)

The above data are of great practical significance, since they open a possibility of controlling friction processes by off-contact loads as effectively as by the contact load.

The studies of friction in Tribo-Fatigue systems are reported in papers [189–195]. On these grounds we formulated the generalized Sosnovskiy – Sherbakov – Komissarov law [24, 52]: *in the general case of a Tribo-Fatigue system the friction force (friction coefficient) is proportional to both the contact and the volumetric load if the latter excites the cyclic stresses* $\pm\sigma_a$ *in the contact zone.*

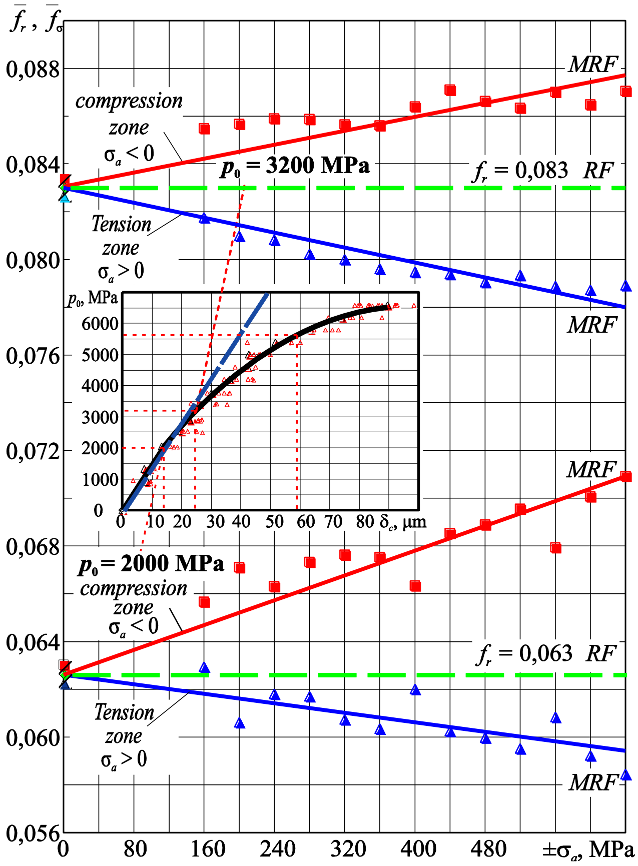

Figure 32 – Back effect: dependence of average values of coefficient of resistance to rolling $\bar{f}_\sigma$ for Tribo-Fatigue system *steel 18KhGT / steel 18KhGT* on amplitude of stresses $\pm\sigma_a$ (dashed lines correspond to value of $\bar{f}_r$ in friction pair) (p_0 is pressure in the contact site center, RF – rolling friction, MRF – mechano-rolling fatigue)

We note that the recognition and attempts of the theoretical description of the fact that off-contact (volumetric) loads can vary the friction characteristics are known and analyzed in the literature (see, e.g., [196]). The generalized law of friction is formulated first in Tribo-Fatigue and presented in the following general form [24]:

$$\tau_\sigma = f_S \frac{1 \pm \mu_p f_F}{1 \pm \chi_p(\sigma_a / p_a)} p_\sigma, \tag{48}$$

where μ_p and χ_p are the hardening-weakening parameters, τ_σ is the specific friction force in the Tribo-Fatigue system.

Table 5 – **Determination of Error of Coefficient of Resistance to Rolling from Experimental Data and Theoretical Basis**

Tribo-Fatigue system	p_0, MPa	Parameter of interaction, a_r		Friction coefficient in Tribo-Fatigue system f_σ				Error, %	
				experimental data		calculation by (47)			
		σ > 0	σ < 0	σ > 0	σ < 0	σ > 0	σ < 0	σ > 0	σ < 0
Steel 18KhGT/ Steel 18KhGT	2000	0.010	0.026	0.060	0.070	0.058	0.067	2.06	5.08
	3200	0.025	0.024	0.076	0.090	0.077	0.089	2.51	1.77
	5600	0.007	0.001	0.087	0.089	0.083	0.095	4.49	6.39

In the simplest case law (48) becomes

$$\tau_\sigma = \tau_w \pm \mu_p \sigma_a \tag{49}$$

where the sign ± takes into account the occurrence of friction in the compression (+) or the tension (–) zones of the shaft. If $\mu_p = 0$, then σ_a does not affect τ_σ. This is possible in the following two cases: 1) no cyclic stresses act in the system; 2) the cyclic stresses are excited but they are not detected in the contact zone. In both cases we consider a common friction pair, for which $\mu_p = 0$ ($\sigma_a = 0$) always. If $\mu_p = 1$, then it means that τ_w and σ_a act on one and the same site and are codirectional. This is the case of, for example, the reciprocal motion of indentor 1 over plate 2 undergoing tension and compression (Figure 33). This is apparently the case of the most pronounced effect of σ_a on τ_σ in the Tribo-Fatigue system. In all intermediate cases ($0 < \mu_p < 1{,}0$) "interaction" between τ_w and σ_a ("to vary" τ_σ) is also intermediate and the growth of μ_p is identical to the intensification of the interaction. It is clear that if τ_w and σ_a are orthogonal (as is shown, for example, in Figures 4 and 6), $\mu_p \neq 0$ since $\sigma_a \neq 0$. And, finally, since the value of σ_a can be either positive or negative, it is possible, in accordance with (49), either to increase or to reduce the value of τ_σ (compared to τ_w).Therefore, *the friction force measured in a Tribo-Fatigue system in the direction of motion can be lesser or higher than the common friction force in a friction pair, or equal to it*, i.e.

$$\tau_a \gtreqless \tau_w \,. \tag{50}$$

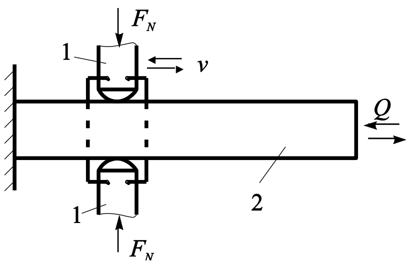

Figure 33 – Schematic of Tribo-Fatigue system, in which F_c and F_S are codirectional

Relation (50) is confirmed theoretically and experimentally [25, 52, 53].

Control over Processes of Wear-Fatigue Damage. If the stress-strain state characterizes the beginning of the life (operation) of a Tribo-Fatigue system and the limiting state characterizes the end of its life, then its evolution is described by the damage state, which varies during the relative motion of the loaded system members. Then the chain of studies in Tribo-Fatigue (TF) is as follows:

$$\begin{gathered} TF: Motion \Rightarrow SSS \Rightarrow V_{ij} \Rightarrow \\ \Rightarrow \Lambda - interaction \Rightarrow LS \left\langle \begin{matrix} DE \\ BE \end{matrix} \right. \Rightarrow Life \end{gathered} \tag{51}$$

Here the following abbreviations are used: SSS is the stress-strain state; V_{ij} is the complex dangerous volume; LS is the limiting state; DE and BE are the direct and back effects, respectively.

As it seen, problem (51) not only fundamental, but also global. It is discussed in more detail in papers [1, 195].

The general contents of the problem of *processes control of wear-fatigue damage* is the following (Figure 34). A Tribo-Fatigue system (TFS) is considered as an *object of control*. Formation of an optimal control program is formulated as an *optimization* problem: $F(\sigma, p) \Rightarrow \min$, $C_0 \Rightarrow \min$, i. e. the technical and economic estimates establish the parameter opt F of optimization. In operation of a TFS a multitude $\{a_j\}$ of parameters a_j of its state is measured. The results of such measurements serve to formulate a current measure $\omega_\Sigma(t)$ of complex wear-fatigue damage that is a function of time t and particular measures of damage $\omega_\sigma, \omega_p, \omega_{ch}, \omega_T$ due to cyclic stresses (the index σ), contact pressure (the index p), processes of electrochemical corrosion (the index ch), and temperature (the index T) in the contact zone between the TFS elements. The TFS state in any moment of operation is rated from the integral parameter $F_t[\sigma, p, \omega_\Sigma(t)]$. Another aim is to correlate (to compare) the optimal (opt F) and current (F_t) values of the integral parameter F. Based on the obtained mismatch ΔF of the parameters opt F and F_t, the problem of synthesis of the dynamic or optimal control $u = \varphi(\Delta F)$ is solved. The physical implementability of TFS control is ensured by an effector (E).

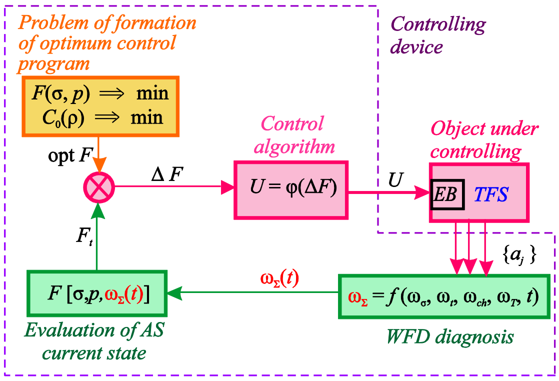

Figure 34 – Contents of problem of management of (dynamic) Tribo-Fatigue system

Of course, the task of controlling the processes of wear-fatigue damage (WFD) is highly intricate, yet very practical: it implies an effective control of functional reliability of the most essential systems of machines – Tribo-Fatigue systems – using the most important criteria of operability, and at the same time it means saving labor, means and materials in the spheres of production and operation.

Figure 35 shows the main *tf-channels*(Tribo-Fatigue channels) *of control* of wear-fatigue damage, the following belong here: (*a*) design and production process parameters ($V_{P\gamma} / V_0$, $S_{P\gamma} / S_k$); (*b*) parameters m_j, $j = 1, 2, \ldots, n$ of composition and structure (the mechano-physico-chemical properties) of materials; (*c*) loading parameters (σ, T, τ_W, D); (*d*) the parameter of time (t); (*e*) the parameters of state (damage) of elements ω_σ, ω_p, ω_T, ω_{ch}; (*f*) the parameters of damage interaction $\Lambda_{\sigma/\tau}$, $\Lambda_{T/M}$, $D_{\sigma,T,\tau}$. Traditionally (in fatigue damage mechanics, Tribology etc.) the parameters of damage and loading parameters of the state shown in Figure 35 are studied separately. The main achievement of Tribo-Fatigue is the methodology of studying Λ-interactions of irreversible damages caused by loads of various nature. Thus, the "backbone" or "spinal cord" (shown in Figure 35 with a wide central arrow) appeared in the general method of calculating the life of Tribo-Fatigue systems.

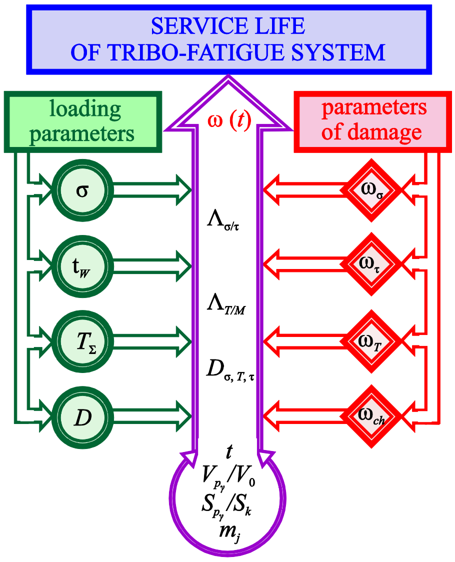

Figure35 –Main *tf*-channels of controlling service life of Tribo-Fatigue systems

Conclusions. From the above it follows that the mechanical-mathematical models formulated on the way from Mechanics of fatigue fracture and Tribology to Tribo-Fatigue and developed in Tribo-Fatigue is a significant evolution of the related mechanical disciplines (Tribology, Solid mechanics, Contact mechanics, Damage mechanics, Mechanics of fatigue fracture etc.).

Thus, in essence, new section of Mechanics is being created. This new section is called *Tribo-Fatigue*. And the necessity to study it at universities has arisen long ago [197–201]. At present the course of Tribo-Fatigue is included in the curriculum of several universities in the Republic of Belarus. We gained the twenty-year experience of teaching in this field and developed fully its scientific and methodical basis [25, 52, 75, 202–205], including a textbook approved by the Ministry of Education of the Republic of Belarus [75].

Some Directions of Further Studies. Let us formulate briefly the basic and promising directions of studies including some problems not discussed in this paper. These important problems are dealt with in works designated in the corresponding references. They include experimental mechanics of Tribo-Fatigue systems [1, 60, 206–215]; design methodologies by Tribo-Fatigue criteria [216–227]; the problems of theory of elasticity supplemented with the consideration of local effects in the area of load application; the problems of contact mechanics supplemented with the effect of various off-contact forces; deformation approach to the analysis of friction in Tribo-Fatigue systems; mechanics of local and wavy damages, i.e. sources of vibration-impact processes at friction (the phenomenon of troppy) [228–233]; the complex of experimental and theoretical studies of diverse Λ-interactions in systems at any scale level; the theories of limiting and, finally, translimiting (transcritical) states of systems and their members [1, 24, 25, 85].

We would like to emphasize clearly that an effective analysis of such intricate phenomenon as wear-fatigue damage and fracture, probably, can not be performed in terms of stresses only. Therefore, in general case it is necessary and worthwhile to apply and develop energy approaches which are successively elaborated in both Mechanics of fatigue fracture and Tribology.

Recent advances in Mechanics of fatigue fracture and Tribology involve the use of the fundamental concepts of thermodynamic entropy. The motivation for this study was a series of experiments and theory of degradation [235]. The development of thermodynamics of fatigue fracture and damage mechanics based on thermodynamics, described in [240, 242, 247, 248, 239, 241, 243]. The recent introduction of the thermodynamics of mechanical fatigue provides a practical use of such data [249, 255]. An interesting application of thermodynamics to the nature of the fatigue of composite materials [256]. Clearly the application of thermodynamics to problems involving wear is also of significant interest [234, 237, 238, 246, 254]. Further experimental and theoretical work on the development of friction and entropy is given in [253, 244, 245].

Now we just note that the outline of the *joined physical discipline, i.e. mechanothermodynamics of systems* [257–262] is already developed, who was possible to formulate the analysis of new results obtained in Tribo-Fatigue (see arrow with a question mark in Figure 1).The notion of the Tribo-Fatigue entropy plays a special role in the algorithmic approach to mechanothermodynamics [239].

Here, in this issue, set out on the path from Tribo-Fatigue to mechanothermodynamics constructed on the basis of energy concepts. In 2015, it is planned, it is to publish a monograph on mechanothermodynamics in English.

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The authors would like to thank Academician, Professor Piotr Vityaz from the National Academy of Sciences of Belarus, Professor Michael Khonsari from the Department of Mechanical and Industrial Engineering at Louisiana State University for the discussions, comments and suggestions which led to the improvement of the paper.

This work was supported by State Research Programs of the Republic of Belarus "Mechanics" [reg. number 20053443, 2003 – 2005]; "New components in mechanical engineering" [reg. number 20044113, 2004 – 2005]; "Mechanics" [reg. numbers 20067084, 20067085 and 20067086, 2006–2010]; "Mechanics, Technical Diagnostics, Metallurgy" [reg. numbers 20111890, 2011–2015; 20111402 and 20111891, 2011–2013]; Belarusian and Russian Foundation for Basic Research (grants №T14R-033 on 23.05.2014 and №14-08-90003-Bel_a (RF)).

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Received 29.06.2015