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MOVEMENT OF A POINT MATERIAL OBJECT IN A MASSIVE RING PLANE

The interaction of a massive ring and a point material object is considered. The case of the point material object location and movement in the plane of the ring is investigated. It is determined that at increasing distance from the ring, the force interaction magnitude decreases. It is shown that there are possible circular trajectories of movement outside the ring, that are similar to motion around a spherical mass distribution.

Keywords: gravity field strength, force interaction, massive ring.

Introduction. In the solar system, some planets have rings. They consist of cosmic dust and ice moving around a planet. Saturn is the first planet to have rings discovered. The nature of the origin of the rings is not completely clear. The Saturn rings were studied by P. Laplace, S. V. Kovalevskaya, D. Maxwell, M. S. Bobrov et al. [1–4].

In [5], the force interaction between a massive ring and a point material object is analyzed. The case of the point material object location in the inner region of the ring plane is considered.

Unlike the spherical shape of a massive object, the ring mass configuration has a more complex force field distribution [6–8]. This paper considers the force interaction between a massive ring and a point material object located in the plane of the ring, as well as possible trajectories of movement.

Determination of a massive ring gravity field strength at points of ring plane. It is assumed that the ring mass has radius R with linear mass density τ. Then the elementary mass dm corresponding to an element of length dl will be $\tau\, dl$ (Figure 1).

According to the law of universal gravitation, the interaction force between two point masses m_1 and m_2 located at a distance ρ from each other is determined by the expression [9–12].

$$F = -\frac{Gm_1 m_2}{\rho^2}, \qquad (1)$$

where G is the gravitational constant.

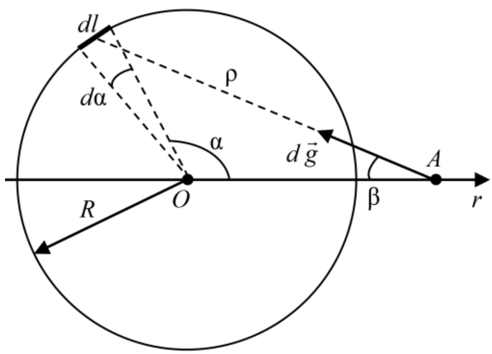

Figure 1 – The scheme of the ring

Then the gravity field strength (the ratio of the interaction force to a unit point mass), created by the elementary mass *dm* at point *A*, located at a distance ρ from it, is determined by the relation

$$dg = -\frac{Gdm}{\rho^2} . \tag{2}$$

The minus sign is caused by the fact that the gravity field strength vector is directed towards the elementary mass.

The element of the circle length *dl* is represented in the form of $Rd\alpha$ (Figure 1). By substituting the value of $\tau R\,d\alpha$ instead of *dm* in (2), there is obtained

$$dg = -\frac{G\tau R d\alpha}{\rho^2} . \tag{3}$$

According to the cosine theorem, it can be written

$$\rho^2 = R^2 + r^2 - 2Rr\cos\alpha . \tag{4}$$

The equation (3) with taking into account the relation (4) turns to the following one

$$dg = -\frac{G\tau R d\alpha}{R^2 + r^2 - 2Rr\cos\alpha} .$$

The gravity field intensity vector $d\vec{g}$ is directed at an angle β to the horizontal axis (see Figure 1). The projection of the gravity field intensity vector onto the horizontal axis is presented in the form

$$dg_{OA} = -\frac{G\tau R d\alpha}{R^2 + r^2 - 2Rr\cos\alpha}\cos\beta . \tag{5}$$

The cosine of the angle β can be expressed in terms of the angle α. From Figure 2, *a* it follows that |*BO*| + |*OA*| = |*BA*|. Then

$$R\cos(\pi - \alpha) + r = \rho\cos\beta .$$

From Figure 2, *b* it follows that |*OB*| +|*BA*| = |*OA*|. Then

$$R\cos\alpha + \rho\cos\beta = r .$$

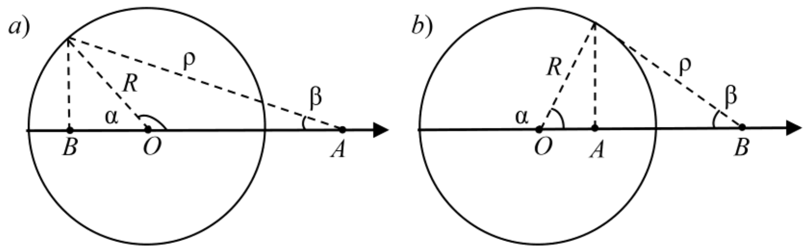

Figure 2 – The schemes for determining the angle β: *a* – for $\pi/2 < \alpha < \pi$; *b* – for $0 < \alpha < \pi/2$

From these relationships it follows that for any values of the angle α

$$\cos\beta = \frac{r - R\cos\alpha}{\rho} . \qquad (6)$$

From expression (5) taking into account (6) there is obtained

$$dg_{OA} = -\frac{(r - R\cos\alpha)G\tau R d\alpha}{\left(R^2 + r^2 - 2Rr\cos\alpha\right)^{1,5}} .$$

Therefore, the gravity field strength created by the entire ring at point *A* can be presented in the following form

$$g = -2\int_0^{\pi} \frac{(r - R\cos\alpha)G\tau R d\alpha}{\left(R^2 + r^2 - 2Rr\cos\alpha\right)^{1,5}} .$$

There can be used a parameter $k = r / R$ (where $k > 1$) and this expression transforms to the form

$$g = -\frac{2\tau G}{R} I(k) ; \; I(k) = \int_0^{\pi} \frac{(k - \cos\alpha)d\alpha}{\left(1 + k^2 - 2k\cos\alpha\right)^{1,5}} . \qquad (7)$$

If the ring mass is *m*, then the gravity field strength is

$$g = -\frac{Gm}{\pi R^2} I(k) = -\frac{Gm}{\pi r^2} k^2 I(k) . \qquad (8)$$

The expressions (7), (8) are similar to the equations obtained in [5, p. 8]. Moreover, in [5] the parameter is used but $k = r / R < 1$.

Combining the obtained results and the results of investigation [5], there is calculated the gravity field strength for all points of the ring plane (except for the case $k = 1$). The calculated values of the integral $I(k)$ for some values of *k* are presented in the Table 1.

Table 1 – **The integral *I*(*k*) values for some values of the parameter *k***

k	*I*(*k*)	*k*	*I*(*k*)	*k*	*I*(*k*)
1,1	11,0504	2	0,978308	20	0,00786874
1,2	5,65895	3	0,381552	30	0,00349357
1,3	3,77654	4	0,206128	40	0,00196442
1,4	2,8031	5	0,129581	50	0,00125701
1,5	2,20497	6	0,0891332	60	0,000872846
1,6	1,79976	7	0,0651146	70	0,00064124
1,7	1,50743	8	0,0496712	80	0,000490931
1,8	1,28704	9	0,0391484	90	0,000387887
1,9	1,1154	10	0,0316538	100	0,000314183

As the strength of the gravity field is known there can be determined the force interaction between a massive ring and a point material object

$$F = -\frac{Gmm_0}{\pi r^2} k^2 I(k) , \qquad (9)$$

where m_0 is the mass of a point material object.

The structure of the equation (9) differs from the structure of the expression (1) by the presence of the factor $k^2 I(k)/\pi$.

Possible circular trajectories. According to (8), the magnitude of the gravity field strength depends on the distance r from the ring center

$$g = \frac{Gm}{\pi r^2} k^2 I(k) . \qquad .$$

At a circular motion with velocity v of a particle (at $k > 1$), the gravity field strength can be written in the following form

$$g = \frac{v^2}{r} .$$

From these equations it follows that the radius of possible circular trajectories

$$r = \frac{Gm}{\pi v^2} k^2 I(k) .$$

The discussion of the results. At k = 10 and k = 100, the values of the $k^2 I(k)$ (expression (8)) are equal to 3.16538 and 3.14183, respectively. When $k \to \infty$ the value of $k^2 I(k)$ tends to π. In this case, the expression (8) takes the form similar to the equation (1). Therefore, at large distances, the field created by the ring and the point mass will be the same.

At k =1.1 and k =1.2, the values of the $k^2 I(k)$ (expression (8)) are equal to 13.370984 and 8.148888, respectively. When $k \to 1_{+0}$ the value of $k^2 I(k)$ tends to ∞. Therefore, the gravity field strength sharply increases at approaching the ring from the outside.

Since there is a minus in the equation (8), and the value of the integral is positive, the force will be directed towards the center of the ring. External particles of matter entering the gravity field outside the ring will be attracted to the ring due to a radially converging force field, which is one of the rings stability factors. Taking into account the results of analysis [5] and the obtained results, there can be generalized that the flying matter particles, approaching the ring, are attracted.

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ДВИЖЕНИЕ ТОЧЕЧНОГО МАТЕРИАЛЬНОГО ОБЪЕКТА В ПЛОСКОСТИ МАССИВНОГО КОЛЬЦА

Рассматривается взаимодействие массивного кольца и точечного материального объекта. Исследуется случай расположения и движения точечного материального объекта в плоскости кольца. Определено, что по мере удаления от кольца величина силового взаимодействия уменьшается. Показано, что возможны круговые траектории движения вне кольца аналогично движению около шарообразного распределения массы.

Ключевые слова: напряженность гравитационного поля, силовое взаимодействие, массивное кольцо.

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