

Анализ полученных результатов позволяет сделать заключение, что при принятых исходных данных увеличение предварительного поджатия пружин, а следовательно, и увеличение первоначального воздушного зазора между скользящими посадочными устройствами и направляющими пути сопровождается значительным увеличением динамических сил.

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### **CAR WITH ELECTROMAGNETIC SUSPENSION EMERGENCY GUIDEWAY LANDING SIMULATION**

Based on German-d'Alembert method the system of dynamic equations for vehicles on magnetic suspension movements has been worked out. Railway car, bogie, electromagnets and bearers during emergency guideway landing vertical and rotary motions have been described with the help of this mathematical model. The calculated dynamic force worked on the car and preliminary springs prepressure dependence has been received.

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### **NON-HOMOGENEOUS TORSION OF THIN-WALLED BEAMS WITH OPEN CROSS-SECTIONS AND LOADED WITH TORSION MOMENT**

Non-homogeneous torsion of thin-walled beams with open cross-sections has been investigated. There is the consideration of three beams with different supports where the load is a torsion moment at the right end. Formulae of the function of the rotation have been deduced. Numerical results are represented in figures.

**1 Introduction.** Thin-walled beams with open cross-section are widely applied in civil and aerospace engineering structural design.

These are two categories of the torsion of these beams – pure (Saint – Venant) torsion and warping (non-homogeneous) torsion. In the case of pure torsion the cross-sections remain flat.

The warping torsion is with an important effect on the static or dynamic response of the beam structure. It is characterized by a warping of the cross-section plane. The axial warping displacements are assumed to depend on the change of the beam's twist angle.

The paper presents a study of warping behavior for thin-walled beam with open cross-section.

**2 Method of solution.** The problem of the warping torsion has been solved in three cases of thin-walled cantilevered beams with open cross-section. They are shown in Figure 1.

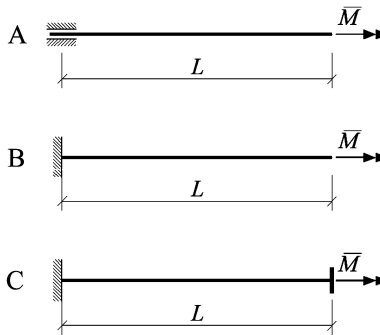


Figure 1 – Schemes of investigated beams

Their lengths are  $L$  and they are loaded with torsion  $M$  moment on free end. The main characteristic of these beams deformation is a function of rotation (the twist angle)  $\varphi(x)$ , where  $x$  is the coordinate of arbitrary cross-section. The differential equation of this function (see reference [1]) is

$$\frac{d^2}{dx^2} \left( EI_{\omega} \frac{d^2 \varphi}{dx^2} \right) - \frac{d}{dx} \left( GI_c \frac{d\varphi}{dx} \right) = 0, \quad (1)$$

where  $E$  – the modulus of elasticity (Young modulus);  $I_{\omega}$  – sectorial moment of inertia;  $G$  – shear modulus;  $I_c$  is the polar moment of inertia.

After the substitution

$$k^2 = \frac{GI_c}{EI_{\omega}} \quad (2)$$

the equation (1) has been transformed in the following form

$$\frac{d^4\varphi}{dx^4} - k^2 \frac{d^2\varphi}{dx^2} = 0. \quad (3)$$

The solution of the equation (1) is presented in the following manner

$$\varphi(x) = C_1 \cosh(kx) + C_2 \sinh(kx) + C_3 kx + C_4, \quad (4)$$

where  $C_1, C_2, C_3, C_4$  – constants.

The other parameters of the problem  $\varphi'(x), B(x), M(x)$  are shown in [2].  $B(x)$  is the bending-torsion bimoment. It is expressed by the formula

$$B(x) = -EI_\omega \frac{d^2\varphi}{dx^2}. \quad (5)$$

In case of warping the torsion moment is presented in the form

$$M(x) = GI_c \frac{d\varphi}{dx} + \frac{dB(x)}{dx}. \quad (6)$$

The first term characterizes the resistance of the beam to the torsion and the second term – its resistance to the cross-section warping.

The function of the rotation  $\varphi(x)$  is presented with the initial four parameters, as it is shown in [2]

$$\varphi(x) = \varphi(0) + \sin(kx) \frac{\varphi'(0)}{k} + (1 - \cosh(kx)) \frac{B(0)}{GI_c} + (kx - \sinh(kx)) \frac{1}{k} \frac{M(0)}{GI_c}. \quad (7)$$

Using the expressions (5) and (6) the parameters  $\varphi'(x), B(x), M(x)$  are presented in the following forms

$$\varphi'(x) = \varphi'(0) \cosh(kx) + B(0) \frac{-k \sinh(kx)}{GI_c} + M(0) \frac{1 - \cosh(kx)}{GI_c}; \quad (8)$$

$$B(x) = -EI_\omega \left[ \varphi'(0) \sinh(kx) + B(0) \frac{-k^2 \cosh(kx)}{GI_c} + M(0) \frac{-k \sinh(kx)}{GI_c} \right]; \quad (9)$$

$$M(x) = M(0). \quad (10)$$

There have been investigated three beams with prevented rotation on the left end  $\varphi(0) = 0$ . Three different conditions have been considered to regard warping of the end-sections.

**For the beam A** the warping are free on the both ends, i.e. no bimoments on the ends. So, the boundary conditions are

$$\varphi(0) = 0; \quad \varphi''(0) = 0 \quad \text{– on the left end of the beam}$$

and  $B(L) = 0 \Rightarrow \varphi''(L) = 0; \quad M(L) = \overline{M}$  – on the right end.

The initial parameters  $B(0)$ ,  $M(0)$  are unknowns in the following linear system:

$$\begin{bmatrix} k \sinh(kL) & \frac{-k \sinh(kL)}{GI_c} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \varphi'(0) \\ M(0) \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{M} \end{bmatrix}. \quad (11)$$

They are expressed in the forms

$$\varphi'(0) = \frac{\overline{M}}{GI_c}; \quad M(0) = \overline{M}. \quad (12)$$

The function of the rotation  $\varphi(x)$  is

$$\varphi(x) = \left[ \frac{(1-k)\sinh(kx)}{k} + kx \right] \frac{\overline{M}}{GI_c}. \quad (13)$$

**For the beam B** the warping is free on the right end and it is constrained on the left end. So, the boundary conditions were

$\varphi(0) = 0$ ;  $\varphi'(0) = 0$  – on the left end of the beam

and  $B(L) = 0 \Rightarrow \varphi''(L) = 0$ ;  $M(L) = \overline{M}$  – on the right end.

The initial parameters  $B(0)$ ,  $M(0)$  are unknown in the linear system

$$\begin{bmatrix} k \cosh(kL) & \sinh(kL) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} B(0) \\ M(0) \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{M} \end{bmatrix}. \quad (14)$$

They are expressed in the forms

$$M(0) = \overline{M}; \quad B(0) = \frac{-\sinh(kL)}{k \cosh(kL)} \overline{M}. \quad (15)$$

The function of the rotation  $\varphi(x)$  is

$$\varphi(x) = \frac{\overline{M}}{GI_c} \left[ -\frac{\sinh(kL)}{k \cosh(kL)} (1 - \cosh(kx)) + kx - \sinh(kx) \right]. \quad (16)$$

**For the beam C** the warping is constrained on both ends. So, the boundary conditions are:

$\varphi(0) = 0$ ;  $\varphi'(0) = 0$  – on the left end of the beam

and  $B(L) = 0 \Rightarrow \varphi''(L) = 0$ ;  $M(L) = \overline{M}$  – on the right end.

The initial parameters  $B(0)$ ,  $M(0)$  are unknown in the linear system

$$\begin{bmatrix} -k \sinh(kL) & 1 - \cosh(kL) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} B(0) \\ M(0) \end{bmatrix} = \begin{bmatrix} 0 \\ \overline{M} \end{bmatrix}. \quad (17)$$

They are expressed in the forms:

$$M(0) = \overline{M}; \quad B(0) = \frac{1 - \cosh(kL)}{k \sinh(kL)} \overline{M}. \quad (18)$$

The function of the rotation  $\varphi(x)$  is

$$\varphi(x) = \frac{\overline{M}}{GI_c} \left[ \frac{1 - \cosh(kL)}{k \sinh(kL)} (1 - \cosh(kx)) + kx - \sinh(kx) \right]. \quad (19)$$

**3 Numerical results.** The non-dimensional parameters have been introduced. They are: non-dimensional parameter, non-dimensional parameter twist angle

$$\varphi_n = \frac{GI_c \varphi(x)}{M l} \quad \text{and non-dimensional parameter } \frac{\varphi}{\varphi_{\max}}.$$

The non-dimensional parameter  $\frac{GI_c \varphi(x)}{M l}$  is used for describing the torsion

flexibility of the beam corresponded to each of these three end conditions. The results are plotted as function of the parameter  $1 \leq kl \leq 10$ . The large warping stiffness is when  $kl \leq 1$  and negligible warping stiffness is when  $kl \geq 10$ . The stiffening effect of the warping constraints for “short” beams is observed.

The of the non-dimensional parameter  $\varphi_n$  and the position of the beam's cross-section C in the case  $kl \leq 4$  dependence drawn in figures 2, a, 3, a and 4, a. For the beams there is a difference for different values. In the cases of a negligible warping stiffness the numerical results are reflected in figures 2, b, 3, b and 4, b. The influence of  $kl$  is expressive at the right half of the beam.

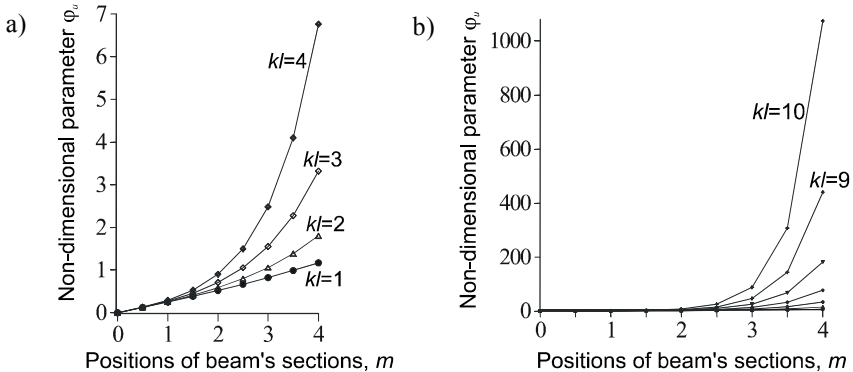


Figure 2 – Non-dimensional parameter  $\varphi_n$  at the length of the beam  $A$  in the cases:

a –  $kl \leq 4$ ; b –  $4 \leq kl \leq 10$

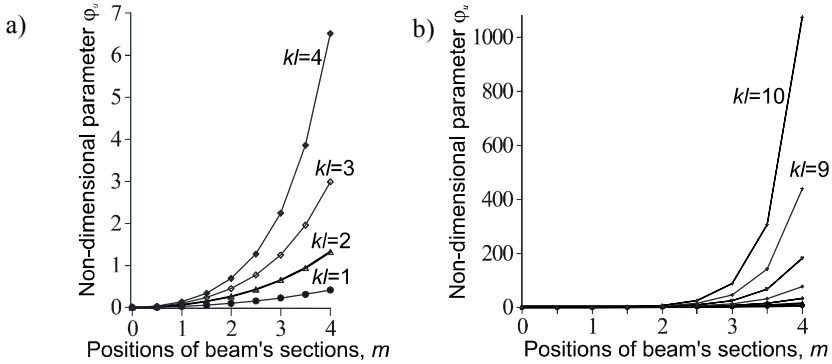


Figure 3 – Non-dimensional parameter  $\varphi_n$  at the length of the beam  $B$  in the cases:  
 a –  $kl \leq 4$  ; b –  $4 \leq kl \leq 10$

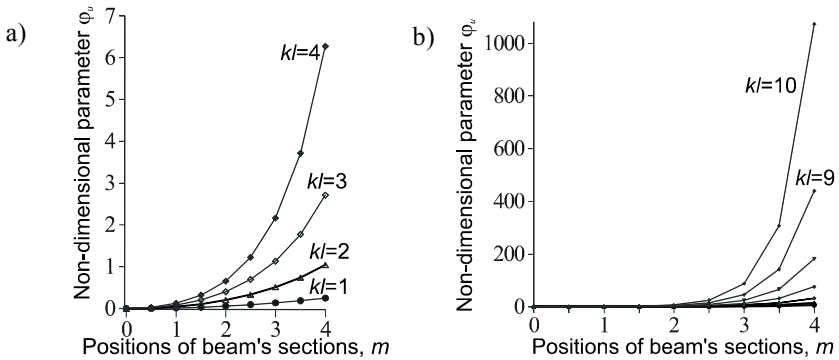


Figure 4 – Non-dimensional parameter  $\varphi_n$  at the length of the beam  $C$  in the cases:  
 a)  $kl \leq 4$  , b)  $4 \leq kl \leq 10$

The influence of the supports of the three beams has been investigated. The numerical results are reflected in figure 5. The values of the non-dimensional parameter  $\varphi_n$  at the length of the three beams are shown. For beams B and C there is a little difference.

The relation  $\varphi / \varphi_{\max}$  for three beams in the case  $kl = 4$  is presented in figure 6. The curves are very close to each other. The supports hasn't got an influence on this non-dimensional parameter. The results of non-dimensional parameter  $\varphi_n$  in the cases  $4 \leq kl \leq 10$  are shown in Table 1.

In the cases of big values of  $kl$  the results for the cases of three beams have been very close. The difference of the supports has got a little influence.

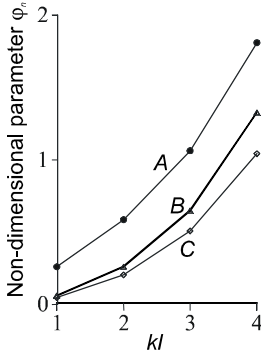


Figure 5 – Non-dimensional parameter  $\varphi_n$  at the length of the beam in the cases of three beams and  $kl = 2$

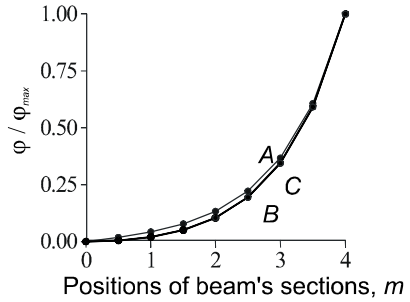


Figure 6 – Non-dimensional parameter  $\varphi / \varphi_{\max}$  at the length of the beam in the cases of three beams and  $kl = 4$

Table 1 – Non-dimensional parameter  $\varphi_n$  at the length of the beam in the cases of three beams and  $4 \leq kl \leq 10$

$kl$	Beam A	Beam B	Beam C
4	6,7642	6,5144	6,2723
5	14,6676	14,4677	14,2730
6	33,1296	32,9629	32,7999
7	76,9776	76,8347	76,6749
8	182,6037	182,4789	182,3339
9	440,0650	439,9539	439,8204
10	1073,8152	1073,7152	1073,5906

**4 Conclusion.** The numerical investigation of the warping torsion of beams with open cross-sections and with three cases of the supports have been presented.

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## **НЕОДНОРОДНОЕ КРУЧЕНИЕ ТОНКОСТЕННЫХ СТЕРЖНЕЙ ОТКРЫТОГО ПРОФИЛЯ, ЗАГРУЖЕННЫХ КРУТЯЩИМ МОМЕНТОМ**

Исследовано неоднородное кручение тонкостенных стержней открытого профиля. Рассматривались три варианта закрепления конца балки, к правому концу которой приложен крутящий момент. Выведены формулы для угла закручивания. Результаты расчетов представлены на графиках.

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## **АНАЛИЗ ПРОЧНОСТИ УПАКОВКИ ШТУЧНОГО ГРУЗА ПОД ДЕЙСТВИЕМ НОРМАТИВНЫХ ЭКСПЛУАТАЦИОННЫХ НАГРУЗОК ПРИ ПЕРЕВОЗКЕ ЖЕЛЕЗНОДОРОЖНЫМ ТРАНСПОРТОМ**

С помощью конечноэлементного моделирования было рассмотрено взаимодействие обвязочной ленты с картонным коробом упаковки штучного груза. Результаты расчетов показали места расположения наибольших напряжений на картоне. Также в работе описаны результаты проведенных экспериментов по определению свойств упаковочных материалов. Полученные результаты позволяют выработать дополнительные меры по обеспечению сохранности груза.

При транспортировке штучных грузов на них действуют переменные нагрузки, обусловленные неравномерностью движения железнодорожного подвижного состава. Наибольшие нагрузки имеют место при соударении вагонов.

Размещать и крепить грузы в крытых вагонах необходимо с учетом обеспечения безопасности движения поездов, производства маневровых и погрузочно-разгрузочных работ, полного использования грузоподъемности или вместимости вагонов, сохранности перевозимых грузов и подвижного состава [1–3]. Вследствие движения вагонов и производства маневровых операций в частях груза и вагона возникают напряжения и деформации, которые могут привести к их повреждению и, следовательно, несохранной перевозке груза.

Наибольшее распространение при анализе напряжений и деформаций в инженерных конструкциях получил метод конечных элементов (МКЭ), на основе которого работает программный комплекс ANSYS.

Решение задач прочности с применением МКЭ во многом зависит от правильного выбора расчетной схемы, аппроксимирующих ее конечных элементов,