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INTEGRALS OF ENERGY AND AREA OF A SYSTEM OF SPINNING TOP

This work shows a model of spinning tops superimposed one over on the other that is represented by a system of two Lagrange gyroscopes, which are connected by elastic hinges, and with two supports on the ends. And we obtain the integrals of energy and area for this system.

Introduction

From ancient times until today, many engineering problems have been studied like systems of connected rigid bodies. A variety of instruments and devices in aviation, maritime, land, and space technology are analyzed as a system of rigid bodies. These types of systems are considered gyrostats, gyroscopic system, manipulators, bodies that spin on a string or a string suspension, wheeled vehicles, and among others. Also the system of rigid bodies has been successfully used for describing the dynamics of complex system like spacecraft [1].

The problem of motion of coupled rigid bodies, even when there are only two bodies, is much more complicated than the classical problem of motion of one body. The first goal of any research is to formulate equations of motion of the object. It would seem that the equations of motion can be apply to the standard methods of analytical dynamics, for example, write down the equations in the form of Lagrange equations. But often these equations are obtained as bulky that their use for solving problems is almost impossible. In this case, we need to write the motion equations in the form that will be the best suited for analysis that represent their characteristics of interest [2].

The work shows a system of two spinning tops superimposed one over the other. This model is represented by two of Lagrange gyroscopes, which are connected by elastic hinges, and with two supports on the ends on inertial and non-inertial systems, and we obtain the integrals of energy and area of the system.

Description

The spinning tops superimposed one over the other, we represent like a system of two identical Lagrange gyroscopes S_i ($n = 1, 2$) connected among themselves in the points O_2 by elastic universal hinges (figure 1). And the point O_1 of the body S_1 is fixed, and the point O_3 of body S_3 moves only along a fixed axis Z going from O_1 to O_3. The l_1 and l_2 are axes of symmetry, and C_2 are the mass centers; h is distance between O_i and O_{i+1} ($i = 1, 2$), and $h/2 = C$.

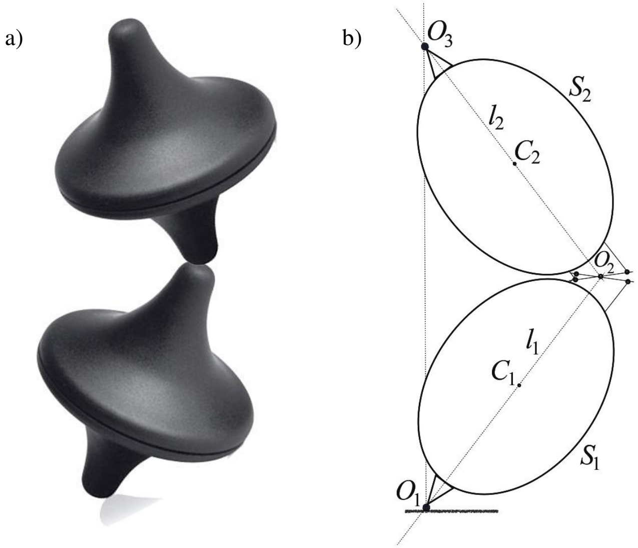

Figure 1 –Spinning tops (a), Lagrange gyroscopes (b)

We introduce a fixed coordinate system O_1XYZ (with unit vectors), whose axis is directed opposite to the gravity vector. And on each body S_i, we bind a moving coordinate system, with a basis $O_i e_i^x e_i^y e_i^z$, the vector $\bar{e}_i^x$ is directed along the symmetry axis of the gyroscope l_i (i = 1, 2). In addition, we assume that the axes of elastic universal joint are directed along the axes of vectors $\bar{e}_1^y$ and $\bar{e}_2^y$. This system has three degrees of freedom.

Mechanical system admits two first integrals: the energy integral and area integral. We write the form of these integrals in the case when the position of the bodies S_i relative to the fixed angles $e_x e_y e_z$ is described by Krilov angles α_i, β_i, γ_i. We consider that the system is in a gravitational field.

Integrals of Energy and Area

The kinetic energy of a system of two Lagrange gyroscopes S_i (i=1,2), can be written as follows [3].

$$T = \frac{1}{2}\int_{V_1} (\bar{\omega}_1 \times \bar{r}_1)^2 \rho dV + \frac{1}{2}\int_{V_2} ([\bar{\omega}_1 \times \overline{O_2 O_3}] + [\bar{\omega}_2 \times \bar{r}_2])^2 \rho dV \,, \qquad (1)$$

where $\bar{r}$ – vector with initial point O_i, aimed at an arbitrary point P_i of the body S_i; ω_i – absolute angular velocity of the body S_i; V_i – volume of the body S_i;

Expressing the quantities in the integrals of (1) through the Krylov angles and integrating, we obtain the following form for the kinetic energy.

$$2T = A_1'(\dot\beta_1^2 + \dot\alpha_1^2 \cos^2\beta_1) + B_1(\dot\gamma_1 - \dot\alpha_1 \sin\beta_1)^2 + A_2(\dot\beta_2^2 + \dot\alpha_2^2 \cos^2\beta_2) + \\ + B_2(\dot\gamma_2 - \dot\alpha_2 \sin\beta_2)^2 + 2m_2c_2h\big[\dot\beta_1\dot\beta_2(\cos\beta_1\cos\beta_2 + \\ + \sin\beta_1\sin\beta_2\cos[\alpha_1-\alpha_2]) + \dot\alpha_1\dot\alpha_2\cos\beta_2\cos\beta_1\cos(\alpha_1-\alpha_2) + \\ + \dot\beta_2\dot\alpha_1\sin\beta_2\cos\beta_1\sin(\alpha_1-\alpha_2) - \dot\beta_1\dot\alpha_2\cos\beta_2\sin\beta_1\sin(\alpha_1-\alpha_2)\big],$$

where A_i – inertia moment main axes (i = 1,2); $A_1' = A_1 + m_2h^2$ – inertia moment; m_i – mass body S_i, h_i – length of point O_i *to* O_{i+1}, c_i – length of center of mass body S_i to point O_{i+1} body S_{i+1}.

Because S_1 and S_2 are connected by a universal hinge, the system has a relation between angles the $\beta_1 = \beta_2 = \beta$, $\alpha_1 = \alpha_2 = \alpha$, we obtain a simple form for the kinetic energy

$$2T = (A_1' + A_2)(\dot\beta^2 + \dot\alpha^2\cos^2\beta) + B_1(\dot\gamma_1 - \dot\alpha\sin\beta)^2 + B_2(\dot\gamma_2 - \dot\alpha\sin\beta)^2 - \\ - 2m_2c_2h[\dot\beta^2(\cos^2\beta - \sin^2\beta\cos 2\alpha) + \dot\alpha^2\cos^2\beta\cos 2\alpha + \\ + 2\dot\beta\dot\alpha(\sin\beta\cos\beta\sin 2\alpha]. \quad (2)$$

For the potential energy of the system we have:

$$P = \mu_1(\bar e_z \cdot \bar e_1^z) + \mu_2(\bar e_z \cdot \bar e_2^z) + P_2(\psi_0^2).$$

where $\mu_1 = g(m_1c_1 + m_2h), \quad \mu_2 = gm_2c_2$;

P_2 is an even differentiable function, its argument ψ_0 is the angle formed by l_1 and l_2. $P_2(0) = 0$, ψ_0 is found through of $\cos\psi_0 = \bar e_1^x \cdot \bar e_2^x$. And by $\beta_1 = \beta_2 = \beta$, $\alpha_1 = \alpha_2 = \alpha$, we obtain

$$P = \mu_1\cos\beta_1\cos\gamma_1 + \mu_2\cos\beta_2\cos\gamma_2 + P_2(\psi_0^2) = \\ = \mu_1\cos\beta\cos\gamma_1 + \mu_2\cos\beta\cos\gamma_2 + P_2(\psi_0^2). \quad (3)$$

The study system is conservative, so it admits the energy integral (2) and (3)

$$E = T + P = \text{const}.$$

As moment of external forces with respect to the vertical, passing through the fixed point O_1, is equal to zero, we have integral of area

$$\bar K \cdot \bar e^z = \text{const}.$$

where K – angular momentum,

$$K = \int_{V_1} \bar r_1 \times (\bar\omega_1 \times \bar r_1)\rho dV_1 + \int_{V_2} (\bar r_2 + \bar s_1) \times (\bar\omega_2 \times \bar r_2 + \bar\omega_1 \times \bar s_1)\rho dV_2. \quad (4)$$

We obtain (4) through of Krilov angles α_i, β_i, γ_i as follows

$$k = A_1(\dot{\alpha}\sin\beta\cos\beta\cos\alpha - \dot{\beta}\sin\alpha) + B_1\cos\beta\cos\alpha(\dot{\gamma}_1 - \dot{\alpha}\sin\beta) +$$
$$+ A_2(\dot{\alpha}\sin\beta\cos\beta\cos\alpha + \dot{\beta}\sin\alpha) + B_2\cos\beta\cos\alpha(\dot{\gamma}_2 - \dot{\alpha}\sin\beta) +$$
$$+ m_2hc_2(-2\dot{\alpha}\sin\beta\cos\beta\cos\alpha - \dot{\beta}\cos 2\beta\sin\alpha) +$$
$$+ m_2h^2[\dot{\alpha}\sin\beta\cos\beta\cos\alpha - \dot{\beta}\sin\beta] = \text{const}.$$

In this work a system of two Lagrange gyroscopes, connected by elastic universal joint is described; it represents a simple model of spinning tops superimposed one over on the other, we obtain integrals of energy and area. We obtained the integrals of area and energy to further study its stationary motions.

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ИНТЕГРАЛЫ ЭНЕРГИИ И ПЛОЩАДЕЙ ДЛЯ СИСТЕМЫ ВОЛЧКОВ

В работе рассмотрена модель наложенных друг на друга волчков, представленная системой двух гироскопов Лагранжа, которые соединены упругим шарниром и закреплены на других концах. Получены интегралы энергии и площадей для этой системы.

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СРАВНИТЕЛЬНЫЙ АНАЛИЗ СПОСОБОВ КРЕПЛЕНИЯ ЯРУСОВ ТРУБ НА ПЛАТФОРМЕ

Рассмотрен способ размещения и крепления труб на железнодорожной платформе. В принятой схеме трубы каждого яруса крепятся в продольном направлении непосредственно к раме платформы. Реквизиты крепления труб содержат натяжные