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FORCE INTERACTION OF A MASSIVE RING AND A MATERIAL POINT LOCATED INSIDE THE RING

The force interaction of a massive ring and a point material object is considered. The case of the location of a point material object in the plane of the ring in its inner region is investigated. It is determined that in the center of the ring the magnitude of the force interaction is minimal and increases with the distance from the center. At approaching directly to the ring, this force sharply increases. It was established that the force acting on a point material object is directed radially from the center.

Keywords: gravitational field strength, force interaction, massive ring.

Introduction. In Newton's theory of gravity, the gravitational mass is the source of the gravitational field. The gravitational field intensity is its power characteristic. The intensity of static gravitational field is determined by the magnitude of the force acting on a resting test body of unit mass.

In the solar system, some planets are surrounded by rings. They consist of cosmic dust and ice orbiting the planet. At first Saturn was discovered as a planet with rings. Galileo was one of the first to suggest that Saturn had "an addition to". This event was in 1610, and in 1655 Christian Huygens first described this "appendage" as a ring surrounding Saturn.

The nature of the origin of the rings is not completely clear. There are two most popular hypotheses for the formation of rings around Saturn and other planets. According to the first, the rings were formed from the substance remnants of a circumplanetary cloud. According to the second theory, the rings appeared as a result of the destruction of a large satellite due to a collision with a meteorite, a large comet or an asteroid. Saturn rings were studied by P. Laplace, S.V. Kovalevskaya, D. Maxwell, M.S. Bobrov [1–4].

The total mass of clastic material in the ring system is estimated as $(1.54 \pm 0.49) \cdot 10^{19}$ kilograms, and it was formed 10^7–10^8 years ago. According to the automatic interplanetary probe Voyager 1 photographs the rings of Saturn consist of hundreds of narrow rings. Subsequently, in the images obtained by the probes, it can be seen that the rings are formed from thousands of rings, alternating with slots. The particles of the rings range in size from 1 centimeter to 10 meters.

In contrast to a gravitating object spherical shape the ring mass configuration has a more complex force field distribution. Therefore, similar problems remain relevant at the present time [5–7]. In this paper, it is considered the force interaction of a massive ring and a point material object (located in the inner region in the plane of the ring).

Force interaction determination for a massive ring and a point material object. The gravitating mass with a certain volume density is assumed to be distributed inside a torus of a small cross section and a large radius *R*. Considering the small cross section of the torus, it can be replaced by a ring of radius *R* with a linear mass density τ. Then the elementary mass *dm*, corresponding to the length element *dl*, is equal to τdl (figure 1).

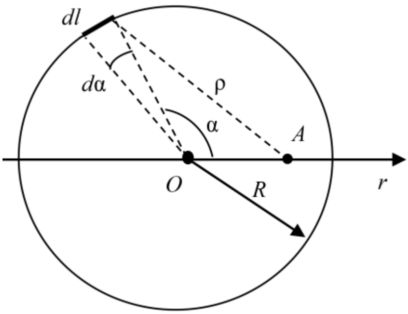

Figure 1 – A ring element and a small material object at point *A*

The gravitational interaction force for the two point masses m_1 and m_2, located at a distance ρ from each other, is determined by the expression [8–10]

$$F = -\frac{Gm_1 m_2}{\rho^2}, \qquad (1)$$

where *G* – the gravitational constant.

Then the intensity of the gravitational field (the ratio of the interaction force to a unit point mass), created by the elementary mass *dm* at point *A*, located at a distance ρ from it, is determined by the relation

$$dg = -\frac{Gdm}{\rho^2}. \qquad (2)$$

The minus sign shows that the gravitational field intensity vector is directed towards the elementary mass.

An element of length *dl* as an elementary part of a circle is considered as $Rd\alpha$ (see figure 1). Substituting the quantity $\tau Rd\alpha$ instead of *dm* in (2), the equation (2) takes the following form

$$dg = -\frac{G\tau Rd\alpha}{\rho^2}. \qquad (3)$$

According to the cosine theorem by the quantity ρ^2 can be replaced by

$$\rho^2 = R^2 + r^2 - 2Rr\cos\alpha \,. \qquad (4)$$

The expression (3), taking into account relation (4), can be written as

$$dg = -\frac{G\tau R d\alpha}{R^2 + r^2 - 2Rr\cos\alpha} \,. \qquad (5)$$

The gravitational field strength vector $d\vec{g}$ is directed at an angle β to the horizontal axis (figure 2).

Considering (5) the projection of this vector onto the axis is

$$dg_{OA} = -\frac{G\tau R d\alpha}{R^2 + r^2 - 2Rr\cos\alpha}\cos\beta. \quad (6)$$

The $\cos\beta$ can be expressed in terms of the angle α from the rectangular triangles shown in figure 3.

From figure 3, *a* it follows that $|BO| + |OA| = |BA|$. Then we can write

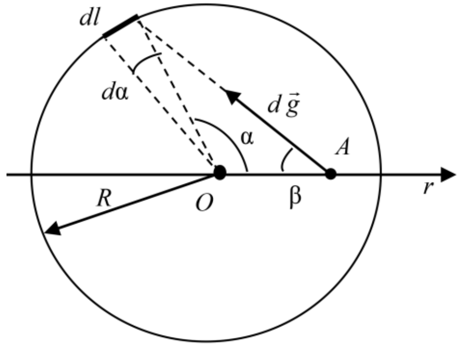

Figure 2 – Direction of a vector $d\vec{g}$ at a point *A*

$$R\cos(\pi - \alpha) + r = \rho\cos\beta \,. \qquad (7)$$

From figure 3, *b* it follows that $|OB| + |BA| = |OA|$. Then we get

$$R\cos\alpha + \rho\cos\beta = r \,. \qquad (8)$$

From figure 3, *c* it follows that $|OA| + |AB| = |OB|$. Then we get

$$r + \rho\cos(\pi - \beta) = R\cos\alpha \,. \qquad (9)$$

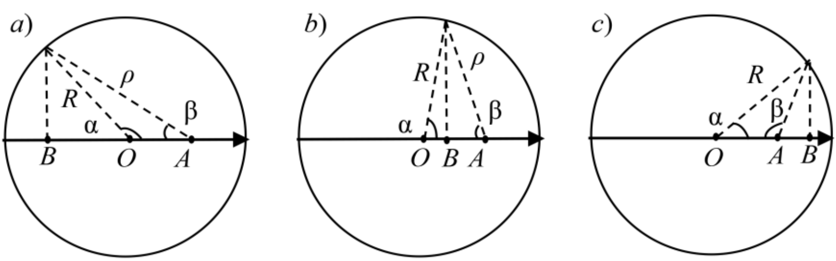

Figure 3 – Various locations of the element *dl*

From relations (7), (8) and (9) it follows that

$$\cos\beta = \frac{r - R\cos\alpha}{\rho} \tag{10}$$

for any angles.

From expression (6), taking into account (10), it can be obtained

$$dg_{OA} = -\frac{(r - R\cos\alpha)G\tau R d\alpha}{\left(R^2 + r^2 - 2Rr\cos\alpha\right)^{1,5}}\,. \tag{11}$$

The intensity of the gravitational field created by the entire ring at the point *A*, taking into account expression (11), is as follows

$$g \;= -2\int_0^{\pi} \frac{(r - R\cos\alpha)G\tau R d\alpha}{\left(R^2 + r^2 - 2Rr\cos\alpha\right)^{1,5}}\,. \tag{12}$$

The relation (12) can be transformed by introducing the parameter $k = \frac{r}{R}$, then

$$g = -\frac{2\tau G}{R} I(k)\,,$$

where the integral $I(k)$ is equal to

$$I(k) = \int_0^{\pi} \frac{(k - \cos\alpha)d\alpha}{\left(1 + k^2 - 2k\cos\alpha\right)^{1,5}}\,.$$

The gravitational field strength can also be expressed in terms of the ring mass

$$g = -\frac{Gm}{\pi R^2} I(k)\,.$$

Knowing the gravitational field intensity, the force interaction of a massive ring and a point material object can be determined by the equation

$$F = -\frac{Gmm_0}{\pi R^2} I(k)\,, \tag{13}$$

where m_0 – is the mass of a point material object.

In this case, the structure of formula (13) differs from the structure of formula (1) by the presence of the factor $I(k) / \pi$. The calculated values of the integral $I(k)$ for some values of k are presented in table 1.

Table 1 – **The calculated values of the integral *I*(*k*)**

k	0	0,1	0,2	0,3	0,4	0,5
$I(k)$	0	–0,158865	–0,328911	–0,523877	–0,763939	–1,08346
k	0,6	0,7	0,8	0,9	0,99	0,999
$I(k)$	–1,55001	–2,32133	–3,87528	–8,63617	–97,6275	–996,502

The integral $I(k)$ in (13) is determined by the parameter k value. When the parameter $k = 0$, a point is in the center of the ring. In this case, the value of the integral becomes equal to 0 and the force is also equal to 0. This is an obvious result due to the symmetry of the ring.

Since there is a minus in formula (13) and the integral value is also negative, the force is positive and, accordingly, it is directed to the right in Figure 1, from the point A in the r axis direction. Thus, the force acting on a point material object is directed radially from the center. This is a significant difference from the spherical mass distribution.

In the ring center the magnitude of the massive ring-point material object force interaction is minimal, and it increases at an increasing distance from the center. At approaching directly to the ring, this force increases sharply, as it follows from the table 1.

Getting into the gravitational field inside the ring, an external substance particles are attracted to the ring due to the radially divergent force field, which is one of the ring stability factors.

The discussion of the results. Analyzing the dependence of the values of the integral $I(k)$, that determines the force magnitude for the points inside the ring, the following conclusions can be presented. Firstly, in the center of the ring, the force interaction magnitude is minimal (absent) and it increases with distance from the center. At point approaching directly to the ring, this force sharply increases. Secondly, the force acting on the point material object from the side of the ring is directed radially from the center, in contrast to the spherical mass distribution. Thirdly, the presence of the radially divergent force field is one of the ring stability factors. Flying particles of substance are attracted at approaching to the ring.

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СИЛОВОЕ ВЗАИМОДЕЙСТВИЕ МАССИВНОГО КОЛЬЦА С РАСПОЛОЖЕННОЙ ВНУТРИ НЕГО МАТЕРИАЛЬНОЙ ТОЧКОЙ

Рассматривается силовое взаимодействие массивного кольца и точечного материального объекта. Исследуется случай расположения такого объекта в плоскости кольца во внутренней его области. Определено, что в центре кольца величина силового взаимодействия является минимальной и по мере удаления от центра она возрастает. При приближении же непосредственно к кольцу эта сила резко возрастает. Установлено, что сила, действующая на точечный материальный объект, направлена радиально от центра.

Ключевые слова: напряженность гравитационного поля, силовое взаимодействие, массивное кольцо.

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