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FLOW STABILIZATION IN THE MULTILAYER VISCOELASTIC TUBES AT NO DISPLACEMENT BOUNDARY CONDITIONS

Stability of the Poiseuille flow through the multilayer viscoelastic anisotropic tube at no displacement boundary condition at the outer surface of the tube is studied for different material parameters and flow regimes. Dependencies of the amplification rate of the most unstable fluid-based mode on the material parameters are obtained. It is shown that the flow can be stabilized by certain choice of viscous and elastic modules of the layers. The range of the parameters is wide enough to provide the flow stability at high variations of Reynold's numbers.

Introduction. Fluid-structure interaction (FSI) is an important factor in the biofluid flows like blood motion in the arteries, capillaries and veins as well as in the flows of other physiological fluids in different ducts and vessels, and flows in distensible tubes of biomedical and technical systems [1–4]. When the flow changes the values of the hydrodynamic pressure, wall shear stress, stress field and wall movement change either. Also there is a strong FSI between fluid and solid structure at their interfaces. FSI and instability of the fluid flows in the compliant ducts have been thoroughly studied in application to the technical fluid-conveying systems as well as to the motion of blood and other biofluids through the vessels or air flow in the airways [5]. When the flow instability causes the flow-limiting phenomena, self-exciting oscillations and noise generation can be observed. The wall oscillations are supported by the energy transfer from the fluid to solid at their interfaces [6].

Flow-induced vibrations of arteries and veins can be detected by acoustic sensors on the human body over the superficial blood vessels. The problem of the pathological and innocent noises separation is an important problem of medical diagnostics [7]. Different problems of the flow instability in the blood vessels as multilayer tubes were studied using the mathematical model of the stationary incompressive axisymmetric flow in a circular three layer viscoelastic tube with no displacement [8] and no stress [9] boundary conditions at the outer surface of the tube. It was found that the unsteady fluid-based modes can be stabilized by the viscosity of the middle layer [8] and by the rigidity of the inner/outer layer [9]. In the both cases the sandwich-type material of the wall helps to stabilize the system. Stabilization of the turbulent flows by special design of the complicated coating is an important problem for many technical and biomedical systems and devices [10]. Firstly, the viscoelastic sandwich-type coatings were proposed for stabilization of internal and external flows based on experiments with dolphins [11]. The unique approach implemented by nature of the dolphin's skin construction has been studied in numerous experiments. Due to its special structure, the skin allows to delay the laminar-turbulence transition and to preserve the laminar flow. That provides enormously low viscous friction of dolphin's body in comparison with manufactured materials with similar geometry. The inspired by nature solution is still widely used for technical and biomedical applications [12, 13]. In this paper the stabilization problem for the layered anisotropic wall is studied and new approaches for the system stabilization at no displacement boundary condition at the outer surface of the wall are proposed.

Problem formulation. Steady flow of viscous Newtonian liquid through a three-layer viscoelastic tube is considered (figure 1). The inner radius R, the length L and the thickness $h = h_1 + h_2 + h_3$ of the tube are given, where h_1, h_2, h_3 are the thicknesses of the undisturbed layers. The outer layer is supposed to be tethered to some rigid areas and in that way the tube can be considered as three-layer viscoelastic coating at the inner surface of a rigid tube.

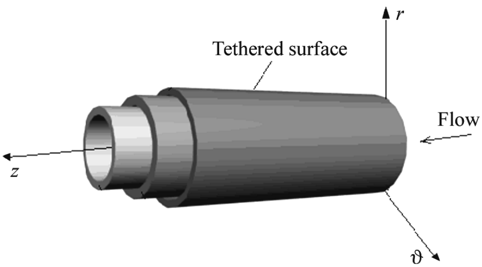


Figure 1 – Coordinate system and 3d model of the tube

The conservation equations for the fluid are the incompressible Navier-Stokes equations

$$\nabla \cdot \vec{v} = 0 \; ;$$

$$\frac{\partial \vec{v}}{\partial t} + \vec{v} \cdot \nabla \vec{v} = -\frac{1}{\rho_f} \nabla p + \frac{1}{\rho_f} \nabla \cdot \hat{\sigma} \, , \tag{1}$$

and the mass and momentum conservation equations for the incompressible wall layers are

$$\nabla \cdot \vec{u}^{(j)} = 0;$$

$$\rho_w^{(j)} \frac{\partial^2 \vec{u}^{(j)}}{\partial t^2} = -\nabla p^{(j)} + \nabla \cdot \hat{\sigma}^{(j)}, \tag{2}$$

where $\vec{v}$ is the fluid velocity, $\vec{u}^{(j)}$ is the wall displacement, ρ_f and $\rho_w^{(j)}$ are the mass densities for the fluid and solid layers, p and $p^{(j)}$ are the hydrostatic pressures, $\hat{\sigma}$ and $\hat{\sigma}^{(j)}$ are the stress tensors for the fluid and the wall layers, j = 1, 2, 3 is the number of the layer.

The constitutive relations for the wall layers are given by viscoelastic Kelvin-Voight model:

$$\tau^{(j)} \frac{\partial}{\partial t} \sigma_i^{(j)} + \sigma_i^{(j)} = A_{ik}^{(j)} \varepsilon_k^{(j)} + \mu_w^{(j)} \frac{\partial}{\partial t} \varepsilon_k^{(j)}, \tag{3}$$

where $A_{ik}^{(j)}$ is the matrix of elasticity coefficients, $\mu_w^{(j)}$ and $\tau^{(j)}$ are viscosities and retardation times for the layers, $\vec{\sigma}^{(j)} = \{\sigma_{11}^{(j)}, \sigma_{22}^{(j)}, \sigma_{33}^{(j)}, \sigma_{23}^{(j)}, \sigma_{13}^{(j)}, \sigma_{12}^{(j)}\}$ is the stress vector, and $\vec{\varepsilon}^{(j)}$ is the similar strain vector. It is supposed the rotations of the infinitesimally small elements can be neglected so the tensor time derivatives in (3) can be replaced by partial derivatives.

The following structure for the matrix of elasticity is accepted:

$$A_{ik}^{-1} = \begin{pmatrix} (E_1)^{-1} & -\nu_{12}(E_2)^{-1} & -\nu_{31}(E_3)^{-1} & 0 & 0 & 0 \\ -\nu_{12}(E_1)^{-1} & (E_2)^{-1} & -\nu_{32}(E_3)^{-1} & 0 & 0 & 0 \\ -\nu_{13}(E_1)^{-1} & -\nu_{23}(E_2)^{-1} & (E_3)^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & G_1^{-1} & 0 & 0 \\ 0 & 0 & 0 & 0 & G_2^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & G_3^{-1} \end{pmatrix}, \tag{4}$$

where E_j, G_j – Young and shear modulus for layer j; ν_{ij} – Poisson ratio.

When the Young and shear modules are distinct and $\nu_{12} = \nu_{21} E_1 / E_2$, $\nu_{13} = \nu_{31} E_1 / E_3$, $\nu_{23} = \nu_{32} E_2 / E_3$ for each layer, the wall layers are orthotropic and possess different properties in the radial, tangential and longitudinal directions. When $E_1 \neq E_2 = E_3$ and $G_1 \neq G_2 = G_3$, the layers are transversely isotropic possessing different properties in the radial direction and in the plane of the cylindrical surface. When $E_{1,2,3} = E$, $\nu_{ij} = \nu$, $G_{1,2,3} = G = \dfrac{E}{2(1+\nu)}$ the layers are isotropic. Note in the case the wall will be still anisotropic in the radial direction when materials of different Young modules are used for adjacent layers.

The boundary conditions are the continuity conditions for velocity and normal stresses at the fluid-solid interface, and the continuity conditions for the displacements and stresses at the and solid-solid interfaces respectively:

$$r = R: \quad \vec{v} = \frac{d\vec{u}^{(1)}}{dt}, \vec{\sigma}_n = \vec{\sigma}_n^1; \tag{5}$$

$$r = R + h_1: \quad \vec{u}^{(1)} = \vec{u}^{(2)}, \vec{\sigma}_n^1 = \vec{\sigma}_n^2; \tag{6}$$

$$r = R + h_1 + h_2: \quad \vec{u}^{(2)} = \vec{u}^{(3)}, \vec{\sigma}_n^2 = \vec{\sigma}_n^3. \tag{7}$$

At the outer surface of the tube the no displacement condition is given at the outer surface

$$r = R + h: \quad \vec{u}^{(3)} = 0. \tag{8}$$

Solution of the problem. The Poiseuille flow in the rigid tube is considered as basic flow $\vec{v}(r, \vartheta, z) = V_P(r)\vec{e}_z$, where $V_P(r)$ is the well-known parabolic profile given by the Poiseuille formula. In the basic flow the wall displacements are supposed to be equal to zero. The solution of the fluid-structure interaction problem (1)–(7) can be found as a superposition of the basic flow and small disturbance in the form of the normal mode:

$$\begin{gathered} \{\vec{v}, p\} = \{V_P(r)\vec{e}_z, P_P(z)\} + \{\vec{v}^\circ, p^\circ\} \cdot e^{st+ikz+in\theta}; \\ \{\vec{u}^{(j)}, p^{(j)}\} = \{0, P^{(j)}\} + \{\vec{u}^{(j)\circ}, p^{(j)\circ}\} \cdot e^{st+ikz+in\theta}, \end{gathered} \tag{9}$$

where $\vec{v}^\circ$, $\vec{u}^{(j)^\circ}$, p°, $p^{(j)^\circ}$ are the amplitudes of the corresponding disturbances, $k = k_r + ik_i$, $s = s_r + is_i$, s_i is the wave frequency, k_r is the wave number, s_r and k_i are spatial and temporal amplification rates, n corresponds to the torsion waves, $P_P(z) = P_{in} - z(P_{in} - P_{out})/L$ is the linear function determined by the inlet P_{in} and outlet P_{out} pressures.

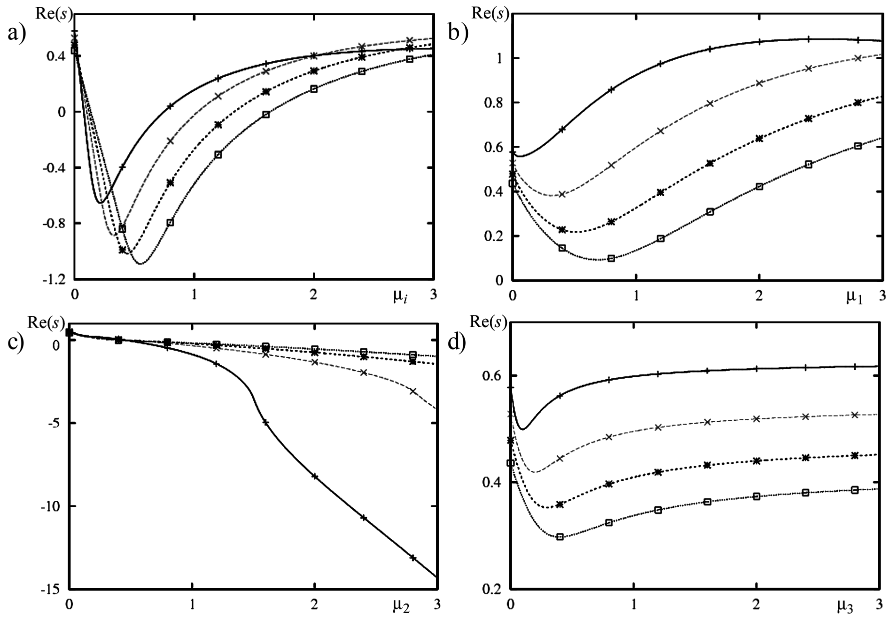
Substitution of (9) in (1)–(3) gives the corresponding system of ODE for the amplitudes $\vec{v}^\circ$, $\vec{u}^{(j)^\circ}$, p°, $p^{(j)^\circ}$ as functions of the radial coordinate. The resulting system has been obtained in [8] and is not reproduced here for brevity. The stable and unstable modes can be computed from the dispersion relation obtained as condition $\det\|\mathbf{M}\| = 0$ where $\mathbf{M}$ is the matrix of coefficients for the resulting system. The solution can be computed numerically. The corresponding procedure was developed for the isotropic layers [14] and then generalized for the anisotropic layers [8].

Results and discussions. Computational results depend on the system geometry and rheology of the fluid and of the viscoeastic solid layers. It is known that the Poiseuille flow in rigid tubes becomes unstable at high Reynold's numbers. Instability caused by any occasional disturbance may produce significant flow changes and transition to turbulence. Flows in the compliant tubes become unstable at relatively small Reynold's numbers that can be demonstrated by the Starling's reservoir [15]. At the no displacement boundary conditions the collapse is impossible and the layers wall can serve as absorber for the energy transferred by small disturbances that leads to stabilization. Different types of resins, polymers, viscoelastic gels, and other materials can be used as the efficient flow stabilizing layered coatings. In [5–10] the rheological parameters of normal human blood and arterial vessel walls have been used for computations of physiological flows. Viscoelastic parameters of biocomposites and of many technical resin-type materials are of the same order of magnitude, so the numerical results may be attributed to both medical and technical flows in ducts.

The following model parameters have been used: $E^*=10^6$ Pa, $\rho_f^* = \rho_w^* = 1000$ kg/m^3, $\mu_w^* = 0.1$ Pa$\cdot$s, $\tau^* = 0.1$ s^{-1}, $P^* = 10^4$ Pa, $R^* = 0.01$ m, $L^* = 0.1$ m, $h^* = 0{,}1\ R^*$.

The behavior of the considered system is complex. It includes many stable fluid-based and solid-based modes with $s_r < 0$ and one or two unstable fluid-based modes with $s_r > 0$, i.e. any small disturbances can grow exponentially providing absolute instability of the system. The unstable modes disappear when stability of the empty three-layer tube is considered but they are taken into account when stability of the liquid column is studied. As it was proposed in [8], the situation when stability of the most unstable mode is increased and s_r becomes smaller and then negative due to variation of some system parameters, first of all, rheological properties of the wall layers. As it was shown in [5–10], the system can be stabilized for quite different Reynold's numbers (Re = 1–1000). It is important to take into consideration the variants of wide and quite narrow diapasons of Re for the stabilization purposes in the systems working at some variety of Re. For instance, in the blood circulation systems the flow regimes vary from Re = 1000–5000 to Re = 0.01–1. Among biomedical devices (blood oxygenators, systems of external blood circulation, hemodialysis, etc.) and technical units (heat and mass exchangers, fluid purification systems, separators of mixtures, etc.) there can be found devices with varying flow regimes.

The influence of the system parameters on the flow stability is studied by the dependencies of temporal amplification rate of the most unstable mode Re(s) at different flow regimes. The dependencies Re(s) on viscosity of the layers are presented in figure 2. The viscosities can be varied for the three layers simultaneously (figure 2, *a*) keeping other rheological properties as characteristic values, i. e. the corresponding non-dimensional parameters are equal to 1. Then the viscosities of two layers are kept as 1, while the viscosity of the first (figure 2, *b*), second (figure 2, *c*) or third (figure 2, *d*) layer takes the values from 1 to 3 (non-dimensional values). The parameters of the wall layers are $\mu_w^{(j)} = 0$ (purely elastic), $\nu_{ik}^{(j)} = 0.4$, $\Xi_1^{(j)} = 20$, $\Xi_{2,3}^{(j)} = 2$, where $\Xi_{1,2,3}^{(j)}$ are non-dimensional Young modulus.



—+— Re = 10; —×— Re = 20; —∗— Re = 30; —□— Re = 40

Figure 2 – The dependence of Re(s) on the viscosity of the three layers μ_i (a), first μ_1 (b), second μ_2 (c) and third μ_3 (d) layers at Re = 10, 20, 30, 40, $\Xi_1^{(j)} = 20$, $\Xi_{2,3}^{(j)} = 2$

As it is shown in figure 2, a, the system can be stabilized by some increase of the all layers viscosities, but the stabilization will be lost at its further increase. A moderate increase up to $\mu_i = 0.1–0.7$ stabilizes the system at Re = 10–50.

In order to stabilize the system at higher Re numbers, it is necessary to increase the viscosities, but the system returns to instability quite fast at $\mu_i > 2$. Any increase in viscosities of the first and third layers (figure 2, b, c) will not stabilize the system and at $\mu_i > 1$ the system becomes more unstable at different Re numbers. The viscosity values of the second layer influence the system stabilization (figure 2, c). When $\mu_2 > 0.5$ the system becomes stable (Re(s) < 0) for Re=10–50. In that way the sandwich-type coatings composed of two elastic layers with one viscoelastic layer in the middle can be used for flow stabilization.

At higher Re the results are qualitatively similar (figure 3). The system can be stabilized by the increase of viscosities of three layers (figure 3, a): from $\mu_i = 0.2 – 2$ at Re = 50 to $\mu_i = 0.05 – 14$ at Re = 200 (figure 3, a). Viscosity of the first (figure 3, b) and third (figure 3, d) layers independently influences the successful stabilizing for Re > 70 and Re > 180 accordingly. The second layer is the most perspective one for modification as it gives reliable stabilizing effect at any Re > 1.

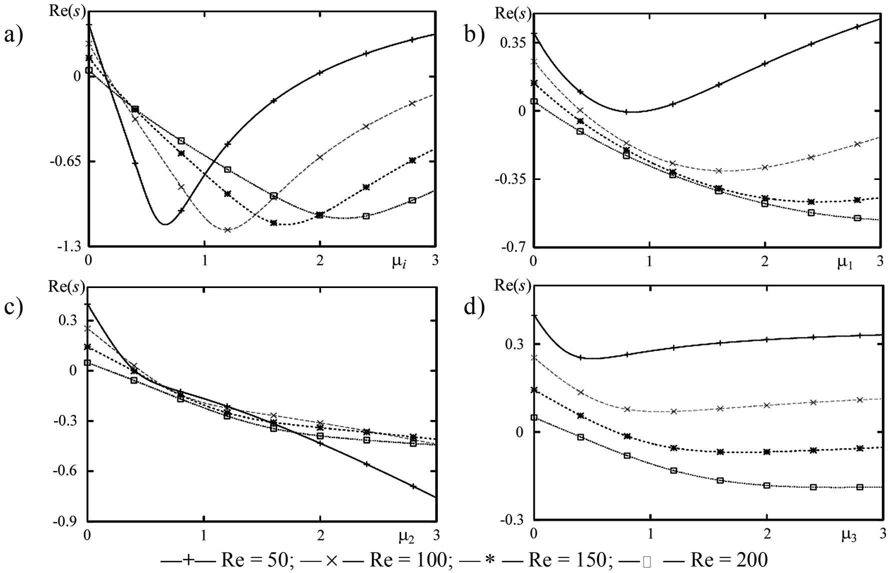
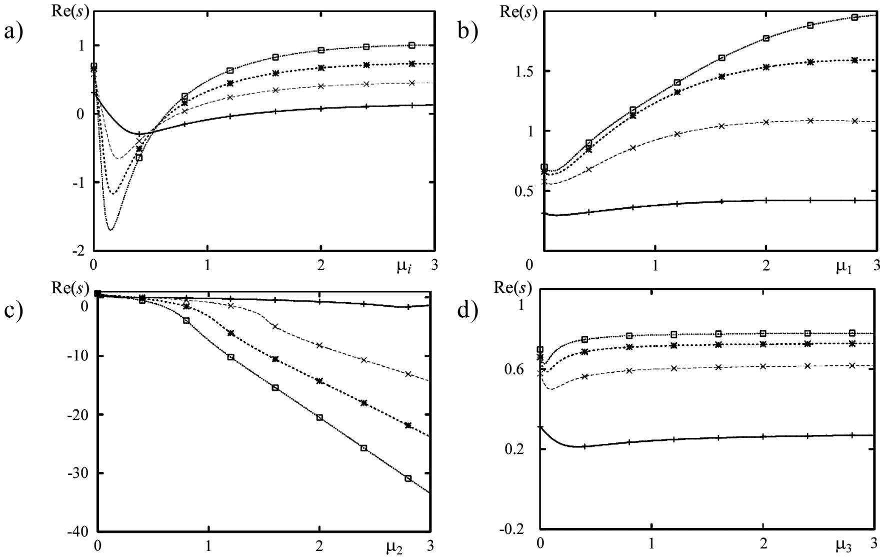


Figure 3 – The dependence of Re(s) on viscosity of the three layers μ_i (a), first μ_1 (b), second μ_2 (c) and third μ_3 (d) layers at Re = 10, 20, 30, 40, $\Xi_1^{(j)} = 2$, $\Xi_{2,3}^{(j)} = 20$

Simultaneous changes of wall elasticity and viscosity also influence the system stability. Some typical results are presented in figure 4 for Re = 10. Any increase of the wall rigidity decreases the stabilizing μ_i diapasons and the reliable values are $\mu_i = 0.2 - 0.5$ (figure 4, a) that corresponds to moderate wall viscosities. When the wall rigidity increases the system may no longer be stabilized by viscosity of the inner (figure 4, b) and outer (figure 4, d) layers, and the middle layer only can be used for stabilization purposes.

Success of the sandwich-type coating at different Young and shear modulus of the layers and the possibility to stabilize the system by the middle layer viscosity can be explained by its damping properties. Rigidity of the inner layer which is in contact with fluid provides better occasional flow perturbations energy transfer into the inner layer, and its damping properties depending on the viscosity and lead to energy dissipation and the perturbations quenching.

Conclusions. The instability of the Poiseuille flow through the multilayer visco-elastic tube strongly depends on the rheological properties of the wall layers. The shear modulus and viscosities of the layers significantly influence the temporal amplification rate of the unstable fluid-based modes. Any increase of the second layer viscosity causes the significant decrease of the amplification rate. The stabilizing sandwich-type coatings composed of two elastic layers with a viscoelastic layer in the middle can damp the flow oscillations at wide variations of the fluid density, viscosity and Reynold's numbers.



—+— G = 5; — × — G = 10; — ∗ — G = 15; —□ — G = 20

Figure 4 – The dependence of Re(s) on viscosity of the three layers μ_i (*a*), first μ_1 (*b*), second μ_2 (*c*) and third μ_3 (*d*) layers at G = 5, 10, 15, 20, Re = 10

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Э. ЧИСТИНА, М. ХАМАДИШ, Н. КИЗИЛОВА

СТАБИЛИЗАЦИЯ ТЕЧЕНИЙ ПО МНОГОСЛОЙНЫМ ВЯЗКОУПРУГИМ ТРУБКАМ ПРИ УСЛОВИИ ЗАКРЕПЛЕНИЯ СТЕНКИ

Исследуется стабилизация пуазейлевского течения жидкости по многослойной трубке при условии закрепления внешней поверхности для различных параметров модели и режимов течения. Проведены численные расчеты инкремента неустойчивости. Показано, что система может быть стабилизирована за счет определенного выбора вязкостей и модулей упругости слоев. Диапазон вычисленных параметров является достаточно широким для стабилизации системы при разных числах Рейнольдса.

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FORMULATION OF THE CONSTRUCTION TOPOLOGICAL OPTIMIZATION PROBLEM FOR RAILWAY STRUCTURES CONSIDERING THE LIMITATIONS ON THE STRENGTH

The main purpose of the paper is the development of the topological structural optimization scientific basis in accordance with the complicated optimization problems of rolling