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PARAMETRIC INSTABILITY OF FERROFLUID LAYER FREE SURFACE IN OSCILLATING MAGNETIC FIELDS

The stability problem is investigated for ferrofluid layer of finite depth and random viscosity in magnetic field consisting of constant and oscillating parts. Using Floquet theory the problem is transformed to the equation for quadratic matrix pencil, determined by amplitude of parametric actions. Neutral stability curves for vertical and horizontal magnetic fields are determined and the difference between these excitation mechanisms of parametric instability is analyzed.

Introduction. The stability problem of fluid free surface in alternating fields is of interest due to prevalence of different parametric actions in technique, f. e. mechanical vibration, temperature fluctuation, sound and electromagnetic fields, etc. Faraday was the first who studied appearance of standing waves on the free surface of fluid layer, subjected to mechanical vibration [1]. As a result of experiments with fluids of different viscosities he found the excitation of parametric resonance on the frequency, equal to half of the principal frequency (subharmonic oscillations) or the same frequency (harmonic oscillations). In [2] it was proposed a method based on the Floquet theory allowing to reduce the given problem to eigenvalue problem for matrices of infinite order and to determine neutral stability curves. The instability of ferrofluid free surface in pure oscillating magnetic field was studied in [3]. The critical value of main amplitudes for cases of vertical, horizontal and rotating of magnetic fields in vertical plane was determined. The constant vertical magnetic field causes the emergence of regularized spatial structures on the ferrofluid free surface when the field magnitude exceeds critical value [4] (Rosensweig instability). The stability of ferrofluid free surface at mechanical vibration and in constant magnetic fields was studied in [5, 6]. The first time of adding the constant part to oscillating magnetic field is described in [7], but the difference from Faraday problem has not been shown, that's why this problem is being still investigated [8]. The presented paper continues research, started in [9], where the stability of unbounded ferrofluid layer in oscillating fields of various natures was studied.

Problem formulation. A horizontally unbounded ferrofluid layer, surrounded by air at the top and limited by nonmagnetic solid plate at the bottom, is considered (see Figure 1). Ω with indexes 1, 2, 3 denotes region, occupied by fluid, air and solid plate respectively.

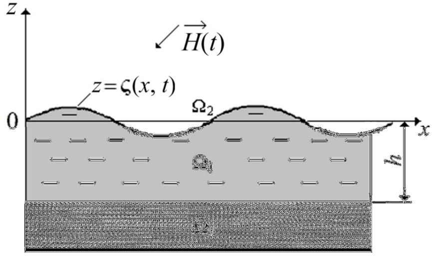


Figure 1 – A sketch of the problem

It is assumed that ferrofluid layer free surface, which is defined as $z = \varsigma(t, x)$, is under parametric action of magnetic field of $\vec{H}(t)$ strength, ς is a perturbation of the free magnetic fluid surface. Magnetizable fluid is considered to be viscous, non-conductive, homogeneous, incompressible and under the most general isotropic magnetization law ($\mu_1 = \mu(\rho, T, H)$, $\mu_2 = \mu_3 = 1$).

Here and further the following notations are introduced: Ψ is a perturbation of the flow function, Φ is a perturbation of the magnetic field strength potential $\left(\vec{H} = \nabla\Phi\right)$, ν and η are kinematic and dynamic viscosities, ρ is a density, μ is magnetic permeability, σ is a coefficient of surface tension, k is a wavenumber, $H_x = \left(\vec{H} \cdot \vec{k}\right)/k$ is a horizontal component of the magnetic field strength, M is a magnetization, T is a temperature, c_v is a heat capacity, $\langle ... \rangle$ is a jump of corresponding values, h is a thickness of the fluid layer.

In the linear approximation fluid motion near equilibrium state is described by the following system of equations [2, 7, 9]:

$$\left(\frac{\partial}{\partial t} - \nu\Delta\right)\Delta\Psi = 0. \tag{1}$$

$$\Delta\Phi_j = 0 \ \text{ in } \ \Omega_j, \quad j = \overline{2,3}. \tag{2}$$

$$\Delta\Phi_1 + c_\infty \vec{H}_\infty \nabla(\vec{H}_\infty \nabla\Phi_1) = 0 \ \text{ in } \ \Omega_1. \tag{3}$$

Boundary conditions for the free surface (at $z = 0$):

$$\eta\left(\frac{\partial^2\Psi}{\partial z^2} - \frac{\partial^2\Psi}{\partial x^2}\right) = 0, \ \frac{\partial\zeta}{\partial t} = -\frac{\partial\Psi}{\partial x}, \tag{4}$$

$$\Phi_1 - \Phi_2 = \frac{(\mu - 1)}{\mu} H_z \varsigma, \tag{5}$$

$$\mu\frac{\partial\Phi_1}{\partial z}-\frac{\partial\Phi_2}{\partial z}+c_\infty H_z(\vec{H}_0\nabla\Phi^{(1)})=-\frac{(\mu-1)}{\mu}H_x\frac{\partial\zeta}{\partial x},\qquad(6)$$

$$\left(\rho\frac{\partial}{\partial t}+\eta\Delta-2\eta\frac{\partial^2}{\partial x^2}\right)\frac{\partial^2\Psi}{\partial x\partial z}=\left(\rho g+\sigma\frac{\partial^2\zeta}{\partial x^2}+\right.$$
$$\left.+\frac{1}{4\pi}\left\langle\frac{(\mu-1)^2}{\mu}H_z\left(\frac{\partial\Phi}{\partial z}-(\vec{H}_x\frac{\partial\zeta}{\partial x})\right)+\frac{(\mu-1)(\mu+c_\infty H_z^2)}{\mu}(\vec{H}_x\nabla\Phi)\right\rangle\right)\frac{\partial^2\zeta}{\partial x^2}.\qquad(7)$$

Boundary conditions for the solid wall (at $z = -h$):

$$\frac{\partial\Psi}{\partial x}=0,\quad \frac{\partial\Psi}{\partial z}=0,\qquad(8)$$

$$\Phi_1=\Phi_3,\quad \mu\frac{\partial\Phi_1}{\partial z}-\frac{\partial\Phi_3}{\partial z}+c_\infty H_z(\vec{H}_0\nabla\Phi^{(1)})=0.\qquad(9)$$

Boundary conditions at infinity ($|z|\to\infty$):

$$\nabla\Phi_j=0,\ \vec{H}_\infty=\left(H_x\ \frac{H_z}{\mu}\right),\ c_\infty=\frac{4\pi}{\mu_\infty H_\infty^2}\left(\frac{\partial M}{\partial H}-\frac{T}{c_v\rho}\left(\frac{\partial M}{\partial T}\right)^2-\frac{M}{H}\right)_\infty.\qquad(10)$$

Formulated in (1) – (10) problem allows to study the instability of free surface for ferrofluid with non-linear magnetization law, taking into account the effects of viscous dissipation and the final depth of the fluid layer.

Analytical solution. Solution of the problem (1)–(10) can be written in the form:

$$\Psi(t,x,z)=\psi(t,z)e^{ikx},\ \Phi(t,x,z)=\phi(t,z)e^{ikx}\ \ \varsigma(t,x)=\xi(t)e^{ikx}.$$

The main qualitative characteristics of these solutions are studying for the linear magnetization law: μ = const, i.e. $c_\infty=0$.

The problem for magnetic field (2), (3) is solved with corresponding boundary conditions (5), (6), (9), (10) and the result is substituted to (7):

$$\left(\rho\frac{\partial}{\partial t}-\eta\frac{\partial^2}{\partial z^2}+3\eta k^2\right)i\frac{\partial\psi}{\partial z}=\left(-\rho k+\sigma k^3-\right.$$
$$\left.-\frac{(\mu-1)^2k^2}{4\pi\left((\mu^2+1)\tan(kh)+2\mu\right)}\left[\frac{H_z^2(\mathrm{t})}{\mu}\left(\mu\tan(kh)+1\right)-H_x^2(\mathrm{t})\left(\tan(kh)+\mu\right)\right]\right)\xi,\qquad(11)$$

where functions $H_z(t)$ and $H_x(t)$ are assumed to be periodic. Using the Floquet theory the hydrodynamic problem (1) is solved with boundary conditions (4), (8) and the result is substituted to (11). Than we get:

$$\sum_{n=-\infty}^{\infty} F_n \xi_n e^{[s+i(\alpha+n)\omega]t} = 0 \tag{12}$$

where
$$F_n = \frac{\nu^2}{q_n \operatorname{cth}(q_n h) - k \operatorname{cth}(kh)} \left\{ k\left(4q_n^2 k^2 + (q_n^2 + k^2)^2\right) - \right.$$
$$\left. - q_n \left[4k^4 + \left(q_n^2 + k^2\right)^2 \right] \operatorname{cth}(q_n h) \operatorname{cth}(kh) + \frac{4q_n k^2 (q_n^2 + k^2)}{\operatorname{sh}(q_n h) \operatorname{sh}(kh)} \right\} - \rho g k - \sigma k^3 +$$
$$+ \frac{(\mu - 1)^2 k^2}{4\pi\left((\mu^2 + 1)\tan(kh) + 2\mu\right)} \left[\frac{H_z^2(\mathrm{t})}{\mu} \left(\mu \tan(kh) + 1\right) - H_x^2(\mathrm{t}) \left(\tan(kh) + \mu\right) \right],$$
$$q_n^2 = k^2 - \frac{s + i(\alpha + n\omega)}{\nu}.$$

Vertical and horizontal components of magnetic field strength are set in the following form:

$$H_z(t) = H_{0z} + m_z \cos\left(n_z \omega t\right),\ H_x(t) = H_{0x} + m_x \cos\left(n_x \omega t\right), \tag{13}$$

where m and ω are the amplitude and the frequency of magnetic field strength relatively.

Expressions (13) are substituted to (12) and a recurrence relation is obtained:

$$A_n \xi_n = \frac{k^2(\mu - 1)}{4\pi\rho\left((\mu^2 + 1)\operatorname{th}(kh) + 2\mu\right)} \left\{ \frac{H_{0z}}{\mu} (\mu \operatorname{th}(kh) + +1)\left[\xi_{n-n_z} + \xi_{n+n_z}\right] m_z + \right.$$
$$\frac{1}{4}(\mu \operatorname{th}(kh) + 1)\left[2\xi_n + \xi_{n-2n_z} + \xi_{n+2n_z}\right] m_z^2 - H_{0x}(\operatorname{th}(kh) + \tag{14}$$
$$\left. + \mu)\left[\xi_{n-n_x} + \xi_{n+n_x}\right] m_x - \frac{1}{4}(\operatorname{th}(kh) + \mu)\left[2\xi_n + \xi_{n-2n_x} + \xi_{n+2n_x}\right] m_x^2 \right\},$$

where

$$A_n = gk + \frac{\sigma k^3}{\rho} - \frac{\nu^2}{q_n \operatorname{cth}(q_n h) - k \operatorname{cth}(kh)} \left\{ \frac{4q_n k^2 (q_n^2 + k^2)}{\operatorname{sh}(q_n h) \operatorname{sh}(kh)} - q_n \left[4k^4 + \right. \right.$$
$$\left. \left. + (q_n^2 + k^2)^2 \right] \operatorname{cth}(q_n h) \operatorname{cth}(kh) + k\left(4q_n^2 k^2 + (q_n^2 + k^2)^2\right) \right\} - \tag{15}$$
$$- \frac{(\mu - 1)^2 k^2 \left(H_{0z}^2 (\mu \operatorname{th}(kh) + 1) - \mu H_{0x}^2 (\operatorname{th}(kh) + \mu) \right)}{4\pi\rho\mu\left((\mu^2 + 1)\operatorname{th}(kh) + 2\mu\right)}.$$

Expression (14) describes general case, when vertical and horizontal parts of magnetic field oscillate with different amplitudes and frequencies. Let's consider particular cases of either vertical or horizontal magnetic fields.

Results and discussions. Magnetic field consisting of constant and oscillating parts (13), leads to a two-frequency parametric action [8, 9], in contrast to the case of mechanical vibration. That's why for vertical or horizontal magnetic field from (14) follows eigenvalue problem for quadratic matrix pencil:

$$\left(m_j^2 C^j + m_j B^j + A^j\right)\xi = 0, \quad j = z, x. \tag{16}$$

where A^z, A^x are diagonal matrices with elements (15) at $H_{0x} = 0$, $H_{0z} = 0$ respectively. B^z, B^x and C^z, C^x are symmetric two- and tridiagonal matrices with elements:

$$B^z_{n,n-1} = B^z_{n-1,n} = -\frac{k^2(\mu-1)^2 H_{0z}(\mu\,\mathrm{th}(kh)+1)}{4\pi\rho\mu\left((\mu^2+1)\,\mathrm{th}(kh)+2\mu\right)},$$

$$B^x_{n,n-1} = B^x_{n-1,n} = \frac{k^2(\mu-1)^2 H_{0x}(\mathrm{th}(kh)+\mu)}{4\pi\rho\left((\mu^2+1)\,\mathrm{th}(kh)+2\mu\right)},$$

$$C^z_{n,n} = -\frac{k^2(\mu-1)^2(\mu\mathrm{th}(kh)+1)}{8\pi\rho\mu((\mu^2+1)\mathrm{th}(kh)+2\mu)}, \quad C^z_{n,n-2} = C^z_{n-2,n} = \frac{1}{2}C^z_{n,n},$$

$$C^x_{n,n} = -\frac{k^2(\mu-1)^2(\mathrm{th}(kh)+\mu)}{8\pi\rho((\mu^2+1)\mathrm{th}(kh)+2\mu)}, \quad C^x_{n,n-2} = C^x_{n-2,n} = \frac{1}{2}C^x_{n,n}.$$

The problem (16) can be reduced to linear eigenvalue problem using vector $\xi^0 = m_j\xi$:

$$\begin{pmatrix} (-C^j)^{-1}B^j & (-C^j)^{-1}A^j \\ I & 0 \end{pmatrix}\begin{pmatrix}\xi^0 \\ \xi\end{pmatrix} = m_j\begin{pmatrix}\xi^0 \\ \xi\end{pmatrix}, \quad j = z, x. \tag{17}$$

where I is identity matrix of the same size as A, B and C. The amplitude of vertical or horizontal field is eigenvalue of the problem (17).

Neutral stability curves are determined in (k, m) by solving eigenvalue problem (18). Matrices A, B and C are cut to size, providing the required accuracy of calculations. The Floquet exponent $\gamma = s + i\alpha$ is fixed on values $s = 0$ and $\alpha = 0$ ($\alpha = 1/2$), corresponding to harmonic (subharmonic) oscillations [2]. For calculations there were used typical for ferrofluid parameters [3]: $\nu = 0{,}1$ P, $\mu = 5$, $\sigma = 30$ erg/cm^2, $\rho = 1$ g/cm^3, $h = 1$ cm, $\omega = 100$ Hz.

Neutral stability curves for pure oscillating vertical magnetic field are shown on Figure 2, *a*. Here (k, m_z) plane is divided into zones ("tongues"), outside (inside). Their parameter values correspond to case of stability (instability). The absolute minimum of neutral curves determined the critical amplitude and wavenumber at which instability occurs.

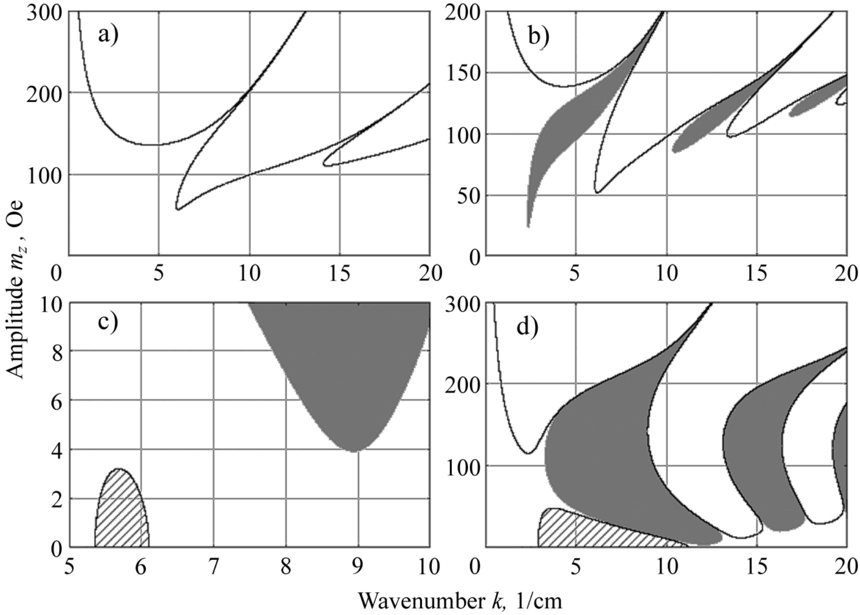
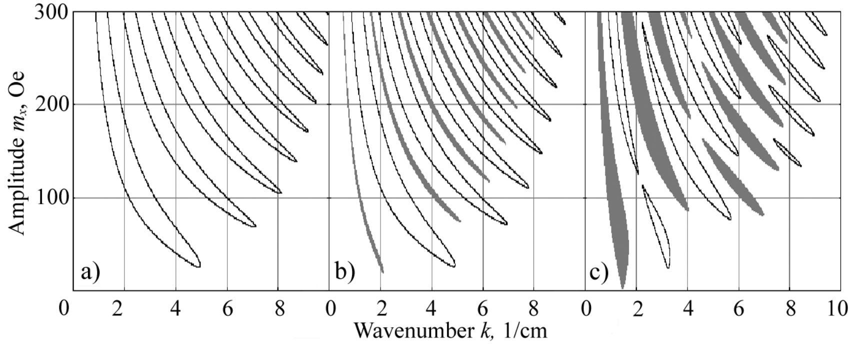


Figure 2 – Neutral stability curves for vertical oscillating magnetic field:
a) $H_{OZ} = 0$, *b*) $H_{OZ} = 20$ Oe, *c*) $H_{OZ} = 90$ Oe, *d*) $H_{OZ} = 100$ Oe

For pure oscillating vertical magnetic field only harmonic instability tongues, formed by neutral stability curves, are the most dangerous. Adding the constant part to vertical oscillating field leads to appearance of bounded regions of subharmonic instability (zones filled in gray). When H_{OZ} increases the subharmonic oscillations become more dangerous (see Figure 2, *b*). When H_{OZ} exceeds critical Rosensweig field H_R, there is an additional (shaded) instability region (see Figure 2, *c* and *d*). In [10] there were experimentally shown, that by adding of oscillating part to constant vertical magnetic field it can lead to Rosensweig instability threshold decrease. But Figure 2, *c*) shows the gap between instability tongues, i.e. Rosensweig instability threshold also can be shifted forward due to parametric action of oscillating magnetic field, similarly to the case of mechanical vibrations [5]. This is possible, if stationary vertical magnetic field doesn't significantly exceed critical value (Figure 2, *d*).

The structure of stability regions for pure oscillating horizontal magnetic field is shown on Figure 3, *a*). The addition of constant part to alternating horizontal field also leads to the transition from harmonic to subharmonic oscillations (Figure 3, *b*). In contrast to the case of vertical field, increase of constant horizontal component leads to the emergence of closed bounded tongues as subharmonic as well as harmonic instability (Figure 3, *c*).



a) H_{OX} = 0, *b*) H_{OX} = 5 Oe, *c*) H_{OX} = 50 Oe

Figure 3 – Neutral stability curves for horizontal oscillating magnetic field

Conclusions. Based on the analytical solution for the linear approximated problem of the magnetizable fluid layer free surface stability in the magnetic fields there were obtained the neutral stability curves for cases of vertical and horizontal magnetic fields. The structural difference for instability regions for these parametric excitations was analyzed.

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ПАРАМЕТРИЧЕСКАЯ НЕУСТОЙЧИВОСТЬ СВОБОДНОЙ ПОВЕРХНОСТИ СЛОЯ НАМАГНИЧИВАЮЩЕЙСЯ ЖИДКОСТИ В ОСЦИЛЛИРУЮЩИХ МАГНИТНЫХ ПОЛЯХ

Исследуется задача устойчивости слоя намагничивающейся жидкости конечной толщины и произвольной вязкости в магнитном поле, которое состоит из постоянной и осциллирующей частей. В линейном приближении с использованием теории Флоке задача сведена к уравнению для квадратичного пучка матриц, где в качестве параметра выступает амплитуда периодического воздействия. Для случаев вертикального и горизонтального магнитных полей определены нейтральные кривые устойчивости и проанализированы отличия между этими механизмами возбуждения параметрической неустойчивости.

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ИСПЫТАНИЯ НА КРУЧЕНИЕ РЕЗИНОМЕТАЛЛИЧЕСКИХ ОПОР

Выполнено экспериментальное исследование деформирования резинометаллических опор, которые предполагается использовать в сейсмозащитных устройствах зданий и сооружений. Получены зависимости крутящего момента и модуля сдвига от угла поворота торцевых сечений опоры при разном количестве ее внутренних металлических пластин.

Наблюдения за повреждениями зданий и сооружений при сильных землетрясениях свидетельствуют о том, что необходимы исследования, связанные с теоретическим анализом и регистрацией параметров крутильных колебаний, которые приводят к разрушениям и потере устойчивости конструкций [1]. При сейсмических колебаниях происходят повреждения или разрушения торцевых стен, изгиб центральных частей протяженных в плане сооружений. Так при разрушительном Спитакском землетрясении 7.12.1988 г. в Армении в зоне бедствия рядом стоящие здания получили развороты и наклоны в противоположных направлениях (рисунок 1), причем смещения и повороты архитектурных памятников и надгробных камней произошли несмотря на хорошее качество цемента.

Анализ показывает, что при сильном сейсмическом воздействии часть здания, расположенная на слабом грунте, испытывает неоднородную осадку, что приводит к появлению горизонтальной составляющей деформации с угловой (вращательной) компонентой. Сдвиг поперечных стен в вертикаль-