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L. A. SOSNOVSKIY, D. Sc., Belarusian State University of Transport, Gomel, S. S. SHERBAKOV, Ph. D., Belarusian State University, Minsk, M. A. ZHURAVKOV, D. Sc., Ministry of Education of the Republic of Belarus, Minsk

NEW SECTION OF PHYSICS

An an attempt to substantiate and formulate the main provisions of combined physical discipline – Mechanothermodynamics. Earlier its principles have been based on the concept of entropy [1]. It is shown that Mechanothermodynamics has combined two sections of Physics – Mechanics and Thermodynamics in order not to compete with each other, as it has been traditionally, but in order to take a fresh look at the evolution of complex systems. Based on the analysis of more than 600 experimental results, the fundamental mechanothermodynamical function of limiting (critical by damage) states of metal and polymer materials was determined.

Introduction. As is known any scientific discipline serves and aims at understanding and describing these or those regularities and features of certain phenomena, situations, events caused by the existence of some real or thinkable objects that manifest themselves their specific properties [1, 2]. Based on the considerations that the study of a new object, as a rule, generates a new scientific discipline as applied to Mechanics, the hierarchy of objects can be built. Successive arrows in Figure 1 [1–4] shows complexity of objects studied by mechanics. The next new objects became multiphase – *mechanothermodynamics system*. For its study, not only sufficient methods of mechanics – both methods are insufficient and only thermodynamics.

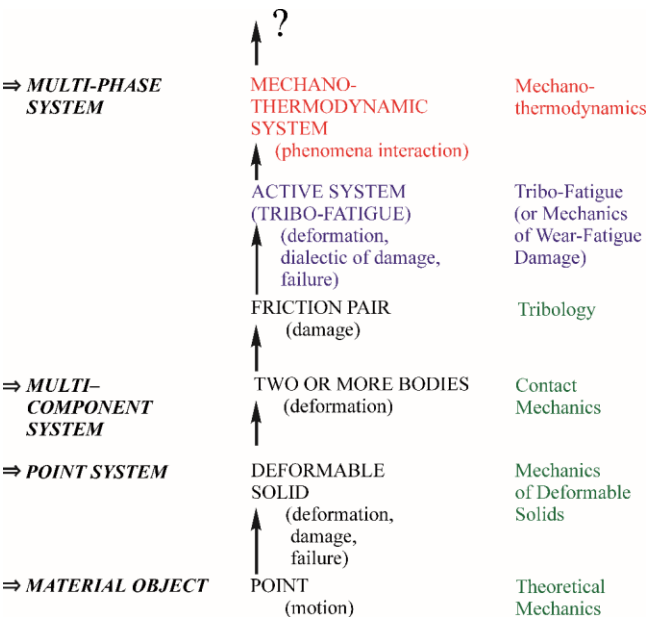


Figure 1 – Simplified hierarchical structure of some objects to be studied in Mechanics: from the simple to the complex

These methods and models addressing coupled problems of both stress-strain states and energy states of the complex systems working under mechanical and thermodynamic loading are discussed in well-known monographs [5–8]. The concepts of entropy and damage are important in order to construct the the models of *mechanothermodynamical system.*

Basic ideas about mechanical behavior of materials under the fracture process are discussed in [9]. The work [10] addresses the main aspects of Damage Mechanics as the branch of Fracture Mechanics and some of its applications. Fundamentals of Physical Mesomechanics of heterogeneous media that develops at the boundary of Continuum Mechanics, Physics of Plasticity and Strength of Materials and studies the stressed and damaged material at linked micro-, meso- and macro levels are given in [11].

Constitutive relations for strain-induced damage in terms of thermodynamic consistency and also applications of Continuum Damage Mechanics to failures of mechanical and civil engineering components in ductile, creep, fatigue and brittle conditions due to thermomechanical loading are considered in [12, 13]. The related problems of constructing theories of vibration and plasticity for steady-state vibrations in elastoplastic bodies are discussed in [14].

The works [15, 16] contain a concise review of basic continuum damage models, micromechanics of damage, kinetics of damage evolution and discuss the areas of further research. General framework for the development of continuum damage models defned by yield and damage surfaces in stress space with consideration of the damage mechanisms (isotropic damage, cracking, etc.) which degrade the stiffness of the material is proposed in [17]. A detailed experimental and theoretical study on the stress-based forming limit criterion during linear and complex strain paths is given in [18]. A thermodynamic framework intended for modelling both friction and non-associated flow for geotechnical materials is presented [19]. The papers [20, 21] are dedicated to modelling of the large strain elastic–plastic deformation behavior of anisotropically damaged ductile metals. The formulation of the elasto-plastic-damage behavior of materials is introduced in [22] within a thermodynamically consistent framework that uses functional forms of hardening internal state variables in both damage and plasticity. Paper [23] proposes a damage theory in terms of kinematic, thermodynamic and kinetic coupling for polycrystalline material

A microscopic damage model considering an ellipsoidal void that is able to change its shape is considered in [24] for mixed-hardening materials. Results of experimental analysis of voids behavior in model materials using X-ray tomography are discussed in [25, 27]. Analythical and scanning electron microscopy based study of void growth and change of shape under large plastic deformation is presented in [28]. Phenomenological representation of anisotropic damage pro-

gression for porous ductile metals with second phases is described through mechanisms of void nucleation, growth and coalescence in [26]. An analytical and computational mesoscopic models for the nucleation and interaction of microcracks near a macrocrack tip based on both the theory of elasticity and the theory of dislocations are presented in [29]. The framework allowing the combination of plasticity and damage models of inelastic behaviour is proposed in [30].

In [1, 31–34], the fundamentals of Mechanothermodynamics are given and two of its principles are formulated: 1) *damage of all things has no conceivable boundaries*, 2) *effective energy fluxes (entropy) that are caused by loads of different nature during irreversible changes in the MTD system have no additivity – they interact dialectically*. In [1], the analysis is made according to the main principles of Tribo-Fatigue [2–4] and Thermodynamics [5] and is based on the concept of *entropy*. In this paper, the similar analysis is made on the basis of the energy representations of Mechanics, Tribo-Fatigue, and Thermodynamics. This has made it possible to reveal and investigate new regularities of the behavior and evolution of the mechanothermodynamical system.

Thermomechanical statement. Consider the statement of thermomechanical problem [5–8] needed for further development of models of energy and entropy states mechanothermodynamical systems.

Energy and entropy descriptions of continuum state of elementary volume dV are of the form [6, 7]

$$\sigma_{ij,j} + \rho f_i = \rho \dot{v}_i, \; i = 1, 2, 3, \tag{1}$$

where the σ_{ij} are the stresses; ρ is the density; the f_i are volumetric forces; the v_i are the velocities.

The *law of conservation of mechanical energy* for the continuum of the volume V, with regard to the repeated index summation rule is obtained by multiplying scalar equation (1) by the velocity vector v_i:

$$\int_V v_i \sigma_{ij,j} dV + \int_V \rho v_i f_i dV = \int_V \rho v_i \dot{v}_i dV. \tag{2}$$

The right hand-side of equation (2) is the *change in the kinetic energy K* of the continuum of volume V:

$$\int_V \rho v_i \dot{v}_i dV = \frac{d}{dt}\int_V \rho \frac{v_i v_i}{2} dV = \frac{d}{dt}\int_V \rho \frac{v^2}{2} dV = \frac{dK}{dt}. \tag{3}$$

Based on the known transformations with regard to *Gauss – Ostrogradsky's the equation for mechnical energy of continuum* [6] is derived

$$\frac{dK}{dt} + \int_V \sigma_{ij}\dot{\varepsilon}_{ij} dV = \int_\Pi \sigma_{ij} l_j d\Pi + \int_V \rho v_i f_i dV, \tag{4}$$

or

$$\frac{dK}{dt} + \frac{\delta u}{dt} = \frac{\delta A}{dt},$$

where ε_{ij} is the strain rate; Π is the continuum surface; the l are the director cones at the continuum surface; $\delta u/dt$ is the power of internal forces; $\delta A/dt$ is the power of internal surfaces and volumetric forces.

The symbol δ in expression (4) is used to underline that the increment in the general case cannot be an accurate differential.

In *the thermomechanical statement the rate of change in internal energy u* [6] is usually given by the integral

$$\frac{du}{dt} = \frac{d}{dt}\int_V \rho u dV = \int_V \rho \dot{u} dV, \tag{5}$$

where $u = \lim\limits_{\Delta m \to 0} \dfrac{u(\Delta m)}{\Delta m}$ is the specific internal energy (internal energy density) of the elementary volume of the mass Δm.

The *rate of heat transfer to the continuum* is expressed as follows:

$$\frac{\delta Q}{dt} = -\int_\Pi c_i l_i d\Pi + \int_V \rho z dV, \tag{6}$$

where c_i is the characteristic of the heat flux per unit area of continuum surface per unit time due to heat conduction; z is the constant of heat radiation per unit mass per unit time.

The *law of change in the energy of thermomechanical continuum* then assumes the form

$$\frac{dK}{dt} + \frac{du}{dt} = \frac{\delta A}{dt} + \frac{\delta Q}{dt}. \tag{7}$$

In (7), the transformation of surface integrals into volume integrals allows the *local form of the energy equation* to be obtained

$$\frac{d}{dt}\left(\frac{v^2}{2} + u\right) = \frac{1}{\rho}\left(\sigma_{ij} v_i\right)_{,j} + f_i v_i - \frac{1}{\rho} c_{i,i} + z. \tag{8}$$

If the scalar product of equation (1) and the velocity vector v_i, is subtracted from equation (8), then the following form of the local energy equation will be obtained:

$$\frac{du}{dt} = \frac{1}{\rho}\sigma_{ij}\dot{\varepsilon}_{ij} - \frac{1}{\rho} c_{i,i} + z = \frac{1}{\rho}\sigma_{ij}\dot{\varepsilon}_{ij} + \frac{dq}{dt}, \tag{9}$$

where dq is the heat flux per unit mass.

According to expression (9), *the rate of change in the internal energy is equal to the sum of the stress power and the heat flux to the continuum*.

As applied to the *thermodynamic system*, two characteristic functions of its state are defined: absolute temperature T and entropy S that can be interpreted as the characteristic of ordering (or chaotic state) of the thermodynamic system. Usually it is assumed that the *entropy possesses the property of additivity*, i.e.,

$$S = \sum_i S_i. \tag{10}$$

In *continuum mechanics* [6, 7], the *specific entropy per unit mass* is considered

$$S = \int_V \rho s dV. \tag{11}$$

The *specific entropy increment ds* can be due to the interaction with the environment (the increment $ds^{(e)}$) or inside the system itself (the increment $ds^{(i)}$) [6, 7]:

$$ds = ds^{(e)} + ds^{(i)}. \tag{12}$$

The increment $ds^{(i)}$ is equal to zero in reversible processes and is greater than zero in irreversible processes.

If the heat flux per unit mass is expressed through dq, then for reversible processes the increment will be

$$Tds = dq. \tag{13}$$

According to the second law of thermodynamics, the rate of change in the total entriopy S of the continuum of the volume V cannot be smaller than the sum of the heat flux through the volume boundary and the entropy produced inside the volume by external sources (*Clausius – Duhem's inequality*) [6, 7]:

$$\frac{d}{dt}\int_V \rho s dV \geq \int_V \rho e dV - \int_\Pi \frac{c_i l_i}{T} d\Pi, \tag{14}$$

where e is the power of local external entropy sources per unit mass. The equality in formula (14) is valid for reversible processes and the inequality – irreversible processes.

Transforming the surface integral into the volume integral in expression (14) can yield the relation for the *rate of the internal entropy production per unit mass*:

$$\gamma \equiv \frac{ds}{dt} - e - \frac{1}{\rho}\left(\frac{c_i}{T}\right)_j \geq 0. \tag{15}$$

In continuum mechanics, it is assumed that the stress tensor can be decomposed into two parts: the *conservative part* $\sigma_{ij}^{(C)}$ for reversible processes (elastic deformation, liquid pressure) and the *dissipative part* $\sigma_{ij}^{(D)}$ for irreversible processes (plastic deformation, liquid viscous stresses):

$$\sigma_{ij} = \sigma_{ij}^{(C)} + \sigma_{ij}^{(D)}. \tag{16}$$

The expression for the *rate of change in energy* (9) can then be presented in the following form:

$$\frac{du}{dt} = \frac{1}{\rho}\sigma_{ij}\dot{\varepsilon}_{ij} + \frac{dq}{dt} = \frac{1}{\rho}\sigma_{ij}^{(C)}\dot{\varepsilon}_{ij} + \frac{1}{\rho}\sigma_{ij}^{(D)}\dot{\varepsilon}_{ij} + \frac{dq}{dt}. \tag{17}$$

If it is assumed that relation (13) is valid for irreversible processes, then the *rate of the total entropy production* is

$$\frac{ds}{dt} = \frac{1}{\rho T}\sigma_{ij}^{(C)}\dot{\varepsilon}_{ij} + \frac{1}{\rho T}\sigma_{ij}^{(D)}\dot{\varepsilon}_{ij} + \frac{1}{T}\frac{dq}{dt}, \tag{18}$$

or

$$\frac{ds}{dt} = \frac{1}{\rho T}\left(\frac{du_M}{dt} + \frac{du_T}{dt}\right) = \frac{1}{\rho T}\left(\frac{du_M^{(C)}}{dt} + \frac{du_M^{(D)}}{dt} + \frac{du_T}{dt}\right).$$

Expression (18) for the rate of a total change of local entropy in the elementary volume of the continuum can be very conveniet in practice.

In view of assumption (10) on the entropy additivity the sum in (18) can be supplemented by other terms considering the internal entropy production in the liquid (gas) volume due to different mechanisms. Similarly, for the continuum volume dV, for example, internal chemical processes can be considered [5]:

$$du = dQ + dA + du_{sub} = TdS - pdV + \sum_1^n \mu_k dN_k, \tag{19}$$

$$dS = \frac{du + pdV}{T} - \frac{1}{T}\sum_1^n \mu_k dN_k. \tag{20}$$

If dV is considered not as a finite volume but as an *elementary volume of continuum*, then changes in its specific energy and entropy based on (17), (19) and (20) can be written in the following differential form:

$$du = \frac{1}{\rho}\sigma_{ij}d\varepsilon_{ij} + dq + \sum_k \mu_k dn_k; \tag{21}$$

$$ds = \frac{1}{\rho T}\sigma_{ij}d\varepsilon_{ij} + \frac{1}{T}dq + \frac{1}{T}\sum_k \mu_k dn_k, \tag{22}$$

where n_k is the number of mols per unit mass.

For the *continuum of the volume* V, expressions (21) and (22) on the basis of relations (5) and (11) will assume the form

$$\begin{aligned} dU = \int_V \rho du dV = \int_V \sigma_{ij} d\varepsilon_{ij} dV + \int_V \rho dq dV + \\ + \int_V \rho \sum_k \mu_k dn_k \; dV; \end{aligned} \tag{23}$$

$$\begin{aligned} dS = \int_V \rho ds dV = \frac{1}{T}\int_V \sigma_{ij} d\varepsilon_{ij} dV + \frac{1}{T}\int_V \rho dq dV + \\ + \frac{1}{T}\int_V \rho \sum_k \mu_k dn_k \; dV. \end{aligned} \tag{24}$$

The introduction of the *chemical entropy component* [the last terms in (21)–(23)] allowed one not only to obtain a more full picture of the continuum state but also to describe self-organization processes that result in initiating stable structures when the heat flux to the continuum is increased.

The above-presented known models of energy and entropy states of continuum (17)–(24), being rather general, do not nevertheless permit one to satisfactorily describe some processes occurring in such a continuum as a deformable solid. However a convenient representation of the additivity of energy and entropy components (11) used, for example, for modeling elastic deformation is not suitable for the description of non-linear processes. The available models do not also allow for the entropy growth due to damageability of solids as a specific characteristic of changes in the structural organization. According to the Tribo-Fatigue concepts [2–32], the damageability is interpreted as any irreversible change in structure, continuum, shape, etc. of a deformable solid that leads to its limiting state. Although, for example, the elasticity limit is not taken into account implicitly during plasticity modeling, the damageability, for example, during mechanical or contact fatigue occurs in the conditions of linear elastic deformation and requires a particular approach for its description with regard to limiting fatigue characteristics of material. The above drawbacks are overcome in the below approach.

Main principles. According to [2, 4, 31], the *mechanothermodynamical* (MTD) *system in the general case represents the thermodynamic continuum with solids distributed (scattered) within it, interacting with each other and with the continuum*. Consider its fragment of limited size $\Omega(X, Y, Z)$ shown in Figure 2. The continuum has a tem-

perature θ and a chemical composition *Ch*. Here are two interacting solid elements (*A and B*) that can move relatively to each other at the contact zone $S(x, y, z)$. Arbitrary mechanical loads applied to one of them (for example, to element *A*) in *x, y, z* coordinate system are transformed into the internal transverse forces Q_x, Q_y, Q_z, longitudinal forces N_x, N_y, N_z and also into the bending moments M_x, M_y, M_z. Element *B* is pressed to element *A* by the loads that are transformed into the distributed normal pressure $p(x, y)$ and the tangential tractions $q(x, y)$. The origin of the coordinates is placed at the point of original contact *O* of the two elements (prior to deformation). It is easy to see that the elements *A* and *B* together form *the Tribo-Fatigue system* [4] which is could be reduced to *the friction pair* [2] *in the absence of internal forces* ($N_i = 0$, $Q_i = 0$, $M_i = 0$, i = *x, y, z*). Thus, the Tribo-Fatigue system is *the friction pair in which at least one of the elements perceives non-contact loads* and, consequently, *undergoes volumetric deformation.* This representation of the MTD system has an advantage that the analysis of the states of a solid and the components of a system can adopt the appropriate solutions known in Mechanics of Deformable Solid, in Contact Mechanics, in Mechanics of Tribo-Fatigue systems (Tribo-Fatigue), and in Tribology.

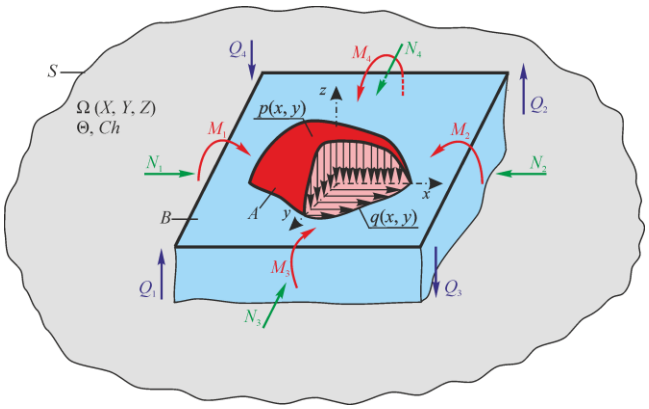


Figure 2 – Scheme of the elementary MTD system

Our main task is to describe *the energy state of the MTD system under the action of mechanical and thermodynamic loads with regard to the environmental influence.*

The energy state of any system is very interesting in itself. However, as applied to the MTD system it is very important to study *its damage* and, as a result, to study *the conditions of reaching the limiting state* (fatigue fracture, wear etc.). Special interest is the analysis of translimiting or supercritical conditions [2].

The main ideas, which are the fundamentals of the given theory, can be formulated with regard to [2–4] as follows.

I Due to the fact that the elements of the MTD system are subject to the loads of different nature – mechanical, thermal and electrochemical, the traditional analysis of their damage and limiting state under the action only of mechanical stresses or strains [35–43, etc.] can be the basis for research. However it appears insufficient and, as a result, is ineffective. This means that there is a need to analyze MTD system states using more general energy concepts.

II Considering that the damage of MTD system solids is determined by mechanical, thermodynamic [44–46, etc.] and electrochemical loads, it is needed to introduce *the generalized representation of its complex damage* that is caused by these loads acting at a time. Call such damage *any irreversible changes in shape, size, volume, mass, composition, structure, continuity and, as a result, physical-mechanical properties of the system elements.* This means the corresponding *changes in the functions of the system as the integrity*.

III The onset and development of complex damage is mainly determined by means of *four particular phenomena: mechanical fatigue, friction, wear, thermodynamic and electrochemical processes.* These phenomena are called particular in the sense that each of them can be realized as independent and separate. This leads to the corresponding energy state and damage in terms of particular (separate) criteria.

IV *In the general case,* all these particular phenomena and processes in the MTD system appear *simultaneously and within one area.* The states of such a system are then caused not by one of any mentioned phenomena but by *their joint (collective) development and, consequently, by their interaction.*

V If *the physical state* of the MTD system is described by means of all input energy u_Σ, then *its damage condition* is determined only by the *effective (dangerous) part* $u_\Sigma^{eff} << u_\Sigma$ that is spent for generation, motion, and interaction of irreversible damages.

VI The effective energy u_Σ^{eff} at the volume strain of solids can be represented by the function of three energy components: *thermal* u_T^{eff}, *force* u_n^{eff}, and *frictional* u_τ^{eff}

$$u_\Sigma^{eff} = F_\Lambda\left(u_T^{eff}, u_n^{eff}, u_\tau^{eff}\right), \qquad (25)$$

where F_Λ takes into account the irreversible kinetic interaction of particular damage phenomena. The components $u_T^{eff}, u_n^{eff}, u_\tau^{eff}$ of the effective energy u_Σ^{eff} have no property of additivity.

VII The processes of electrochemical (in particular, corrosion) damage of solids can be taken into consideration by introducing the parameter $0 \le D_{ch} \le 1$ and can be studied, for example, as *electrochemical damage under the influence of temperature* ($D_{T(ch)}$), *stress* ($D_{\sigma(ch)}$), and *friction corrosion* ($D_{\tau(ch)}$). So function (25) takes the form

$$u_\Sigma^{eff} = F_\Lambda\left(u_{T(ch)}^{eff}, u_{n(ch)}^{eff}, u_{\tau(ch)}^{eff}\right). \qquad (26)$$

VIII *The generalized criteria of the limiting (critical) state* is represented by the condition when the effective energy u_Σ^{eff} reaches its limiting value – *critical quantity* u_0 in some area of limited size – in *the dangerous volume* of the MTD system.

IX The energy u_0 is considered to be *a fundamental constant for a given material*. It shouldn't depend on testing conditions, input energy types, damage mechanisms.

X By the dangerous volume is understood *the 3D area* $V_{ij} \subset V_0$ of the deformable solid (V_0 is its working volume) with *the critical state of material at all its points,* of which it consists.

XI In the general case, the limiting (critical) state of the MDT system is reached not due to a simple growth of effective energy components and, hence, due to the accumulation of irreversible damages that are caused by separate actions (loads of different nature), but as a result of *their dialectical interaction, whose direction is characterized by the develop-*

ment of spontaneous phenomena of hardening-softening of materials in the given operating or testing conditions. In such a way, taking into consideration function (26), *the hypothesis of the limiting (critical) state of the MTD system* can be represented in the following general form

$$\Phi(u^{eff}_{\sigma(ch)},\, u^{eff}_{\tau(ch)},\, u^{eff}_{T(ch)},\, \Lambda_{n\backslash k\backslash l},\, m_k,\, u_0) = 0, \qquad (27)$$

where m_k, $k = 1, 2, \ldots$, are some *characteristic properties (hardening-softening)* of contacting materials, $\Lambda_{k\backslash l\backslash n} \gtreqless 1$ are *the functions (parameters) of dialectic interactions of effective energies (irreversible damages) that are caused by loads of different nature.* This means that at $\Lambda_k > 1$, the damage increase is realized, at $\Lambda_l < 1$ – its decrease, and at $\Lambda_n = 1$ – its stable development.

XII Taking into consideration Item III, *from the physical viewpoint, hypothesis* (27) *should be multi-criterion,* i.e., it should describe *not only the states of the system as the integrity but its separate elements in terms of different criteria of performance loss* (wear, fatigue damage, pitting, corrosion damage, thermal damage, etc.). In particular cases, it is possible to reach the corresponding limiting (critical) states in terms of one or two, three or several criteria at a time.

XIII Reaching the limiting state

$$u^{eff}_{\Sigma} = u_0 \qquad (28)$$

means *the full loss of the integrity of the MTD system,* i.e., *of all its functions.* At the same time *damageability* of its elements

$$0 < \psi^{eff}_u = u^{eff}_{\Sigma} / u_0 \qquad (29)$$

reach *the critical value*

$$\psi^{eff}_u\left(\psi_{\sigma(ch)}, \psi_{\tau(ch)}, \psi_{T(ch)}, \Lambda_{k\backslash l\backslash n}, m_k\right) = 1. \qquad (30)$$

XIV If $t = t_0$ is the time of origination of the system and $T_{\oplus}$ is the time of reaching the limiting state, then *the failure time of its functions* corresponds to the *relative lifetime (longevity)* $t/T_{\oplus} = 1$. But *the system lifetime* T_* as *the material object* is longer than its lifetime as the functional integrity ($T_* >> T_{\oplus}$) since at the time moment $t > T_{\oplus}$ the process of *degradation – disintegration* is realized by forming a great number of remains, pieces, fragments, etc. This process develops under the influence of not only possible mechanical loads but mainly of the environment – up to *the system death as the material object* at the time moment $t = T_*$. The system death means *its complete disintegration into an infinitely large number of ultimately small particles* (for example, atoms). *The translimiting existence of the system as a gradually disintegrating material object* can then be described by the following conditions

$$\psi^{eff}_u \to \infty, \qquad (31)$$

$$d_{\psi} \to 0, \qquad (32)$$

where d_{ψ} is the average size of disintegrating particles and the natural relation $\psi^{eff}_u(d_{\psi})$ should exist between ψ_{Σ} and d_{ψ}. Then *the condition for the system death* is

$$t/T_* = 1. \qquad (33)$$

XV The particles of the "old system" disintegration are not destructed but are spent for the formation and growth of a number of "new systems". This is *the* essence *of the MTD system evolution hysteresis*.

Energy theory of damage and limiting states. First specify function (25).

To determine the effective energy, consider *the work of internal forces in the elementary volume dV of Tribo-Fatigue systems* (*A, B* in Figure 2). In the general case, the differential of the work of the internal forces and the temperature dT_{Σ} can be written with regard to the rule of disclosing the biscalar product of *the stress and strain tensors* σ and ε:

$$du = \sigma_{ij} \cdot\cdot d\varepsilon_{ij} + kdT_{\Sigma} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} & \sigma_{xz} \\ \sigma_{yx} & \sigma_{yy} & \sigma_{yz} \\ \sigma_{zx} & \sigma_{zy} & \sigma_{zz} \end{pmatrix} \cdot\cdot \begin{pmatrix} d\varepsilon_{xx} & d\varepsilon_{xy} & d\varepsilon_{xz} \\ d\varepsilon_{yx} & d\varepsilon_{yy} & d\varepsilon_{yz} \\ d\varepsilon_{zx} & d\gamma_{zy} & d\varepsilon_{zz} \end{pmatrix} +$$
$$+kdT_{\Sigma} = \sigma_{xx} d\varepsilon_{xx} + \sigma_{yy} d\varepsilon_{yy} + \sigma_{zz} d\varepsilon_{zz} + \sigma_{xy} d\varepsilon_{xy} + \sigma_{xz} d\varepsilon_{xz} +$$
$$+\sigma_{yz} d\varepsilon_{yz} + kdT_{\Sigma}; \qquad (34)$$

here k is the Boltzmann constant.

We proceed from the idea that in the general case, according to [2, 4], the main role in forming wear-fatigue damage is played by the normal and shear stresses that cause the processes of *shear (due to friction) and tear (due to tension-compression).*

In this case, it is reasonable to divide the tensor T into two parts: T_{τ} is *the tensor of friction-shear stresses,* or, briefly, *the shear tensor* and T_{σ} is the tensor of normal stresses *(tension-compression),* or, briefly, *the tear tensor*. So in (28), the tear part T_{σ} and shear part T_{τ} of the tensor T will be set as:

$$du = \sigma^{(V,W)}_{ij} \cdot\cdot d\varepsilon^{(V,W)}_{ij} + kdT_{\Sigma} = \left(\sigma_n{}^{(V,W)} + \sigma_{\tau}{}^{(V,W)}\right) \cdot\cdot d\varepsilon^{(V,W)}_{ij} +$$
$$+kdT_{\Sigma} = \sigma_n{}^{(V,W)} \cdot\cdot d\varepsilon^{(V,W)}_{ij} + \sigma_{\tau}{}^{(V,W)} \cdot\cdot d\varepsilon^{(V,W)}_{ij} + kdT_{\Sigma} \cdot =$$
$$= du_n + du_{\tau} + du_T. \qquad (35)$$

According to Items III and IV, the tensors T and E should be represented as follows:

$$\sigma_{ij} = \sigma^{(V,W)}_{ij} = \sigma_{ij}\left(\sigma^{(V)}_{ij}, \sigma^{(W)}_{ij}\right),$$
$$\varepsilon_{ij} = \varepsilon^{(V,W)}_{ij} = \varepsilon_{ij}\left(\varepsilon^{(V)}_{ij}, \varepsilon^{(W)}_{ij}\right). \qquad (36)$$

Here the stress and strain tensors with the superscript V are caused by the action of volume loads (the general cases of 3D bending, torsion, tension-compression) and those with the superscript W – by the contact interaction of the system elements. Expression (35) with regard to (36) can be given as follows:

$$du = \sigma^{(V,W)}_{ij} \cdot\cdot d\varepsilon^{(V,W)}_{ij} + kdT_{\Sigma} = \left(\sigma_n{}^{(V,W)} + \sigma_{\tau}{}^{(V,W)}\right) \cdot\cdot d\varepsilon^{(V,W)}_{ij} +$$
$$+kdT_{\Sigma} = \sigma_n{}^{(V,W)} \cdot\cdot d\varepsilon^{(V,W)}_{ij} + \sigma_{\tau}{}^{(V,W)} \cdot\cdot d\varepsilon^{(V,W)}_{ij} + kdT_{\Sigma} \cdot =$$
$$= du_n + du_{\tau} + du_T. \qquad (37)$$

In the case of the linear relationship between the stresses and strains, expression (36) will assume the form

$$\sigma_{ij} = \sigma^{(V,W)}_{ij} = \sigma^{(V)}_{ij} + \sigma^{(W)}_{ij} =$$
$$= \begin{pmatrix} \sigma^{(V)}_{xx} + \sigma^{(W)}_{xx} & \sigma^{(V)}_{xy} + \sigma^{(W)}_{xy} & \sigma^{(V)}_{xz} + \sigma^{(W)}_{xz} \\ \sigma^{(V)}_{yx} + \sigma^{(W)}_{yx} & \sigma^{(V)}_{yy} + \sigma^{(W)}_{yy} & \sigma^{(V)}_{yz} + \sigma^{(W)}_{yz} \\ \sigma^{(V)}_{zx} + \sigma^{(W)}_{zx} & \sigma^{(V)}_{zy} + \sigma^{(W)}_{zy} & \sigma^{(V)}_{zz} + \sigma^{(W)}_{zz} \end{pmatrix}, \qquad (38)$$

$$\varepsilon_{ij} = \varepsilon_{ij}^{(V,W)} = \varepsilon_{ij}^{(V)} + \varepsilon_{ij}^{(W)} = \begin{pmatrix} \varepsilon_{xx}^{(V)} + \varepsilon_{xx}^{(W)} & \varepsilon_{xy}^{(V)} + \varepsilon_{xy}^{(W)} & \varepsilon_{xz}^{(V)} + \varepsilon_{xz}^{(W)} \\ \varepsilon_{yx}^{(V)} + \varepsilon_{yx}^{(W)} & \varepsilon_{yy}^{(V)} + \varepsilon_{yy}^{(W)} & \varepsilon_{yz}^{(V)} + \varepsilon_{yz}^{(W)} \\ \varepsilon_{zx}^{(V)} + \varepsilon_{zx}^{(W)} & \varepsilon_{zy}^{(V)} + \varepsilon_{zy}^{(W)} & \varepsilon_{zz}^{(V)} + \varepsilon_{zz}^{(W)} \end{pmatrix}, \quad (39)$$

and (37) will be as follows

$$du = u = \frac{1}{2}\sigma_{ij} \cdot\cdot \varepsilon_{ij} + kT_\Sigma = \frac{1}{2}\left(\sigma_{ij}^{(V)} + \sigma_{ij}^{(W)}\right) \cdot\cdot \left(\varepsilon_{ij}^{(V)} + \varepsilon_{ij}^{(W)}\right) +$$

$$+kT_\Sigma = \frac{1}{2}\left[\left(\sigma_n^{(V)} + \sigma_n^{(W)}\right) + \left(\sigma_\tau^{(V)} + \sigma_\tau^{(W)}\right)\right] \cdot\cdot \left(\varepsilon_{ij}^{(V)} + \varepsilon_{ij}^{(W)}\right) +$$

$$+kT_\Sigma =$$

$$= \frac{1}{2}\left[\begin{pmatrix} \sigma_{xx}^{(V)} + \sigma_{xx}^{(W)} & 0 & 0 \\ 0 & \sigma_{yy}^{(V)} + \sigma_{yy}^{(W)} & 0 \\ 0 & 0 & \sigma_{zz}^{(V)} + \sigma_{zz}^{(W)} \end{pmatrix} + \begin{pmatrix} 0 & \sigma_{xy}^{(V)} + \sigma_{xy}^{(W)} & \sigma_{xz}^{(V)} + \sigma_{xz}^{(W)} \\ \sigma_{yx}^{(V)} + \sigma_{yx}^{(W)} & 0 & \sigma_{yz}^{(V)} + \sigma_{yz}^{(W)} \\ \sigma_{zx}^{(V)} + \sigma_{zx}^{(W)} & \sigma_{zy}^{(V)} + \sigma_{zy}^{(W)} & 0 \end{pmatrix}\right] \cdot\cdot$$

$$\cdot\cdot \begin{pmatrix} \varepsilon_{xx}^{(V)} + \varepsilon_{xx}^{(W)} & \varepsilon_{xy}^{(V)} + \varepsilon_{xy}^{(W)} & \varepsilon_{xz}^{(V)} + \varepsilon_{xz}^{(W)} \\ \varepsilon_{yx}^{(V)} + \varepsilon_{yx}^{(W)} & \varepsilon_{yy}^{(V)} + \varepsilon_{yy}^{(W)} & \varepsilon_{yz}^{(V)} + \varepsilon_{yz}^{(W)} \\ \varepsilon_{zx}^{(V)} + \varepsilon_{zx}^{(W)} & \varepsilon_{zy}^{(V)} + \varepsilon_{zy}^{(W)} & \varepsilon_{zz}^{(V)} + \varepsilon_{zz}^{(W)} \end{pmatrix} + kT_\Sigma. \quad (40)$$

From (40) it is seen that the tear part σ_n of the tensor σ is the sum of the tear parts of the tensors at the volume strain $\sigma_n^{(V)}$ and the surface load (friction) $\sigma_n^{(W)}$, whereas the shear part σ_τ is the sum of the shear parts $\sigma_\tau^{(V)}$ and $\sigma_\tau^{(W)}$. This means *the vital difference* of the generalized approach to the construction of the criterion for the limiting state of the MTD system.

From total energy (40), its effective part is separated according to Items V and VIII with regard to [2, 3]. To do this, introduce the coefficients of appropriate dimensions $A_n(V)$, $A_\tau(V)$ and $A_T(V)$ that determine the fraction of the absorbed energy

$$du_\Sigma^{eff} = \Lambda_{M\setminus T}(V)\left\{\begin{array}{l} \Lambda_{\tau\setminus n}(V)\left[\begin{array}{l} A_n(V)\sigma_n \cdot\cdot d\varepsilon_{ij} + \\ +A_\tau(V)\sigma_\tau \cdot\cdot d\varepsilon_{ij} \end{array}\right] + \\ +A_T(V)kdT_\Sigma \end{array}\right\} \quad (41)$$

or

$$du_\Sigma^{eff} = \Lambda_{M\setminus T}(V)\left\{\begin{array}{l} \Lambda_{\tau\setminus n}(V)\left[\begin{array}{l} A_n(V)du_n + \\ +A_\tau(V)du_\tau \end{array}\right] + \\ +A_T(V)du_T \end{array}\right\} \quad (42)$$

where $\Lambda_{M\setminus T}(V)$ and $\Lambda_{\tau\setminus\sigma}(V)$ are the functions of interaction between energies of different nature. The subscript $\tau\setminus n$ means that the function Λ describes the interaction between the shear (τ) and tear (σ) components of the effective energy, and the subscript $M\setminus T$ means that the function Λ describes the interaction between the mechanical (M) and thermal (T) parts of the effective energy. That fact that the coefficients A can be, generally speaking, different for different points of the volume V, enables one to take into account the inhomogeneity of environment.

Taking into consideration (42), criteria (27) can be specified with no regard to the environmental influence:

$$\Lambda_{M\setminus T}(V)\left\{\Lambda_{\tau\setminus n}(V)\left[du_n^{eff} + du_\tau^{eff}\right] + du_T^{eff}\right\} = u_0. \quad (43)$$

In the case of the linear relationship between the stresses and strains, expressions (41) and (42) will be as follows:

$$u_\Sigma^{eff} = \Lambda_{M\setminus T}(V)\left\{\Lambda_{\tau\setminus n}(V)\left[\frac{1}{2}A_n(V)\sigma_n \cdot\cdot \varepsilon_{ij} + \right.\right.$$

$$\left.\left. + \frac{1}{2}A_\tau(V)\sigma_\tau \cdot\cdot \varepsilon_{ij}\right] + A_T(V)kT_\Sigma\right\}, \quad (44)$$

or

$$u_\Sigma^{eff} = \Lambda_{M\setminus T}(V)\left\{\Lambda_{\tau\setminus n}(V)\left[\begin{array}{l} A_n(V)u_n(V) + \\ +A_\tau(V)u_\tau(V) \end{array}\right] + A_T(V)u_n(V)\right\} =$$

$$= \Lambda_{M\setminus T}(V)\left\{\Lambda_{\tau\setminus n}(V)\left[u_n^{eff}(V) + u_\tau^{eff}(V)\right] + u_T^{eff}(V)\right\}. \quad (45)$$

With regard to expression (36), criterion (43) can be represented as follows:

$$u_\Sigma^{eff} = \left\{\begin{array}{l} \left[\begin{array}{l} u_n^{eff}(\sigma_n^{(V,W)}, \varepsilon_n^{(V,W)}) + \\ +u_\tau^{eff}(\sigma_\tau^{(V,W)}, \varepsilon_\tau^{(V,W)}) \end{array}\right] \Lambda_{n\setminus\tau} + \\ +u_T^{eff} \end{array}\right\} \Lambda_{T\setminus M} = u_0. \quad (46)$$

When the time effects should be taken into consideration, criterion (46) will assume the form:

$$u_{\Sigma t}^{eff} = \int_0^t \left\{\begin{array}{l} \left[\begin{array}{l} u_n^{eff}(\sigma_n^{(V,W)}, \varepsilon_n^{(V,W)}, t) + \\ +u_\tau^{eff}(\sigma_\tau^{(V,W)}, \varepsilon_\tau^{(V,W)}, t) \end{array}\right] \Lambda_{n\setminus\tau}(t) + \\ +u_T^{eff}(t) \end{array}\right\} \Lambda_{T\setminus M}(t)\, dt = u_0. \quad (47)$$

Thus, expression (45) is the concretization of function (25) and formula (46) is the concretization of criterion (27) for that case when the environmental influence is not taken into account.

Criterion (27) in the form of (46) and (47) says: *when the sum of interacting effective energy components caused by the action of force, frictional, and thermal (thermodynamic) loads reach the critical (limiting) quantity* u_0, *the limiting (or critical) state of the MTD system (of both separate elements of the system and the system as the integrity) is realized. Physically, this state is determined by many and different damages.*

The fundamental character of the parameter u_0 has been mentioned above. According to [47–59], the parameter u_0 will be interpreted as *the initial activation energy of the disintegration process.* It is shown that the quantity u_0 approximately *corresponds both to the sublimation heat for metals and crystals with ionic bonds* and *to the activation energy of thermal destruction for polymers*:

$$u_0 \approx u_T.$$

On the other hand, the quantity u_0 is determined as *the activation energy for mechanical fracture:*

$$u_0 \approx u_M.$$

In such a way, the energy u_0 can be considered to be *the material constant*:

$$u_0 \approx u_M \approx u_T = \text{const}. \tag{48}$$

Taking into consideration the physical-mechanical and thermodynamic representations of the processes of damage and fracture [48, 49, 51], write down (48) in the following form

$$u_M = s_k \frac{\sigma_{th}}{E} \frac{C_a}{\alpha_V} = u_0 = kT_S \ln \frac{k\theta_D}{h} = u_T, \tag{49}$$

where s_k is the reduction coefficient, σ_{th} is the theoretical strength, E is the elasticity modulus, C_a is the atom heat capacity, α_V is the thermal expansion of the volume, k is the Boltzmann constant, T_S is the melting point, θ_D is the Debye temperature, h is the Planck constant. According to (49), it can be taken approximately [48]

$$u_0 \approx \varepsilon_* \frac{C_a}{\alpha_V}, \tag{50}$$

where $\varepsilon_* \approx 0.6$ is the limiting strain of the interatomic bond. Calculations according to (50) are not difficult. Methods of experimental determination of the quantity u_0 have also been developed [49].

From equality (49) it follows that u_0 is the activation energy of a given material, which is by the order of magnitude equal to 1–10 eV per one particle or molecule (~10^2–10^3 kJ/mol), i.e., *the value that is close to the energy of interatomic bond rupture in the solid* [52]. Its level doesn't depend on how the rupture is reached – mechanically, thermally or by their simultaneous action. In [49], it is possible to find the tables containing the u_0 values for different materials.

From (49) it is possible to find the *thermomechanical constant of the material* [2]

$$\frac{\sigma_{th}}{T_S} = E \frac{\alpha_V k}{C_a} \ln \frac{k\theta_D}{h} = \theta_\sigma. \tag{51}$$

The constant θ_σ characterizes *the strength loss per 1 K*.

Criterion (46) is written in the absolute values of physical parameters – the values of the effective and critical energy components. This criterion can be easily made dimensionless by diving it by the quantity u_0. Then it can be represented *in terms of irreversible (effective) damage*

$$\psi_u^{eff} = \frac{u_\Sigma^{eff}}{u_0} = 1. \tag{52}$$

It is clear that *the local (at the point) energy measure of damage* ψ_u^{eff} is within the range

$$0 \le \psi_u^{eff} \le 1, \tag{53}$$

or in detailed form

$$0 \le \psi_u^{eff} = \frac{\Lambda_{T\setminus M}}{u_0} \left\{ \left[\begin{matrix} u_n^{eff}(\sigma_n^{(V,W)}, \varepsilon_n^{(V,W)}) + \\ + u_\tau^{eff}(\sigma_\tau^{(V,W)}, \varepsilon_\tau^{(V,W)}) \end{matrix} \right] \Lambda_{n\setminus\tau} + \right.$$
$$\left. + u_T^{eff} \right\} \le 1. \tag{54}$$

According to (54), it is also possible to determine *particular energy measures of damage*

$$0 \le \psi_n^{eff} = \frac{u_n^{eff}\left(\sigma_n^{(V,W)}, \varepsilon_n^{(V,W)}\right)}{u_0} \le 1, \tag{55}$$

$$0 \le \psi_\tau^{eff} = \frac{u_\tau^{eff}\left(\sigma_\tau^{(V,W)}, \varepsilon_\tau^{(V,W)}\right)}{u_0} \le 1, \tag{56}$$

$$0 \le \psi_T^{eff} = \frac{u_T^{eff}}{u_0} \le 1, \tag{57}$$

due to the effective energies of different nature that are determined by the force (the subscript n), frictional (the subscript τ), and thermodynamic (the subscript T) loads, respectively. Now criterion (52) can be written in dimensionless form

$$\psi_u^{eff} = \left[\left(\psi_n^{eff} + \psi_\tau^{eff}\right)\Lambda_{n\setminus\tau} + \psi_T^{eff}\right]\Lambda_{M\setminus T} = 1. \tag{58}$$

According to (58), *the limiting state of the MTD system is reached when the sum of interacting damages* ($0 < \psi < 1$) *at mechanical and thermodynamic loads is equal to 1*. Criterion (46) in form (58) is convenient because all damage measures are dimensionless and are within the same range $0 \le \psi \le 1$.

Since numerous and infinite actions, as well as the interaction effects of physical damages of many types (dislocations, vacancies, non-elastic deformations, etc.) cannot be described and predicted exactly, when analyzing the MTD system, one introduces the concept of *the interaction of dangerous volumes* [2] that contain a real complex of damages (defects generated by the action of the corresponding fields of stresses (strains)). According *to the statistical model of the deformable solid with the dangerous volume* [53], such a volume should depend on the geometric parameters of the solid responsible for its working volume V_0, on the distribution function parameters $p(\sigma_{-1})$ and $p(\sigma)$ of the durability limit σ_{-1} and the effective stresses σ considering both the effective stress probabilities P and γ_0 and gradients G_σ:

$$V_{P_\gamma} = F_V\left[p(\sigma_{-1}),\ p(\sigma),\ G_\sigma,\ V_0,\ P,\ \gamma_0,\ \vartheta_V\right]. \tag{59}$$

Here ϑ_V describes how the limiting durability is influenced by the body shape and the scheme of body loading during fatigue tests.

Thus, *the dangerous volume can serve as the equivalent of the complex of damages*, as its value is proportional, in particular to the level of effective stresses and, hence, to the number (concentration) of defects (damages).

The boundary between the volumes of dangerous and safe, as it follows from expression (59), is generally blurred and probabilistic in nature. As the damage probability P of the solid increases, the dangerous volume V_{P_γ} is growing. At a given value of P the volume can vary depending on the confidence probability γ_0. It means that at P = const

$$V_{P_{\gamma\,\min}} \le V_{P_\gamma} \le V_{P_{\gamma\,\max}}, \tag{60}$$

if $\gamma_{\min} \le \gamma_0 \le \gamma_{\max}$. Here $\gamma_{\min}, \gamma_{\max}$ form the permissible range. If it is accepted that γ_0 = const, then the dangerous volume will have a single value associated with the damage probability *P*.

Scattered damage within the dangerous volume is characteristic not only for the so-called smooth bodies but also for the elements with the structural *stress concentrators* [53]. Figure 3 demonstrates several microcracks on the sharp cut (rounding radius *r* = 0.5 mm, the theoretical stress concentration factor α_n = 8 in Figure 3, *a*) and on the flat cut (*r* = 2 mm, α_n = 2.55 in Figure 3, *b*) and also two fatigue cracks at a distance of 25 mm from each other at a fillet connection from the crankshaft journal to its web (*r* = 18 mm, α_n = 3.2 in Figure 3, *c*); the crankshaft journal diameter is 360 mm.

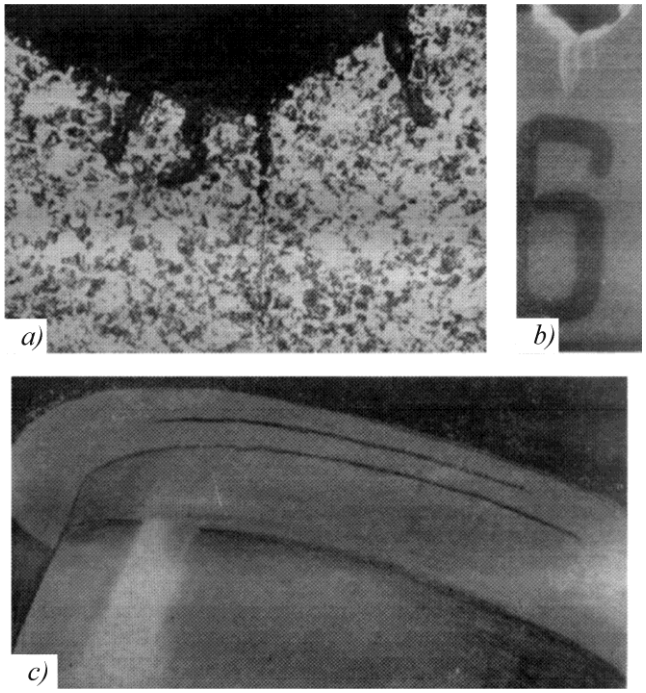


Figure 3 – Fatigue microcracks in the zones of stress concentrators (*L. A. Sosnovskiy*)

Thus, if in the uniaxial stress state, the stress distribution $\sigma(x, y, z)$ in *x*, *y*, *z* coordinates is known, then *the dangerous volume should be calculated by the formula*

$$V_{P\gamma} = \iiint\limits_{\sigma(x,y,z)>\sigma_{-1\min}} dxdydz\,, \tag{61}$$

where $\sigma_{-1\min}$ being the lower boundary of the range of the durability limit σ_{-1} statistical distribution is such that if $\sigma_{-1} < \sigma_{-1\min}$, then *P* = 0.

From expression (61) it follows that *the generalized condition for fatigue fracture* is of the form

$$V_{P\gamma} > 0 \tag{62}$$

with some probability *P* under the confidence probability γ_0.

If

$$V_{P\gamma} = 0\,, \tag{63}$$

then the fatigue damage cannot occur physically (because in this case, $\sigma < \sigma_{-1\min}$); hence, (63) is *the generalized condition of non-fracture.*

The methods for calculation of dangerous volumes V_{ij} for friction pairs and Tribo-Fatigue systems are developed similar to (59)

$$V_{ij} = V_{ij}\left(\sigma_n^{(V,W)},\, \sigma_\tau^{(V,W)},\, \sigma_{\lim}^{(V,W)},\, G_{\sigma_{ij}},\, V_0, P,\, \gamma_0\right) \tag{64}$$

and outlined in [4, 54–56]. Here $\sigma_{\lim}^{(V,W)}$ is the limiting stress based on the assigned criterion of damage and fracture.

Further, the following dimensionless characteristics of damage can be introduced: *integral energy damage within the dangerous volume*

$$\Psi_u^{eff}(V) = \iiint\limits_{\psi_u^{eff}(dV)\ge 1} \frac{u_\Sigma^{eff}}{u_0} dV \tag{65}$$

and *the average energy damage (at each point of the dangerous volume)*

$$\overline{\Psi}_u^{eff}(V) = \frac{1}{V_u} \iiint\limits_{\psi_u^{eff}(dV)\ge 1} \frac{u_\Sigma^{eff}}{u_0} dV. \tag{66}$$

The accumulation of energy damage in time within the dangerous volume is described by the formulas

$$\Psi_u^{eff}(V,t) = \int_t \iiint\limits_{\psi_u^{eff}(dV)\ge 1} \frac{u_\Sigma^{eff}}{u_0} dVdt \tag{67}$$

$$\overline{\Psi}_u^{eff}(V,t) = \int_t \frac{1}{V_u} \iiint\limits_{\psi_u^{eff}(dV)\ge 1} \frac{u_\Sigma^{eff}}{u_0} dVdt. \tag{68}$$

Having used (63)–(68), the MTD system damage can be described and analyzed using the most general representations – the energy concepts with regard to the influence of numerous and different factors taken into account by (59), including *the scale effect*, i.e., the changes in the size and shape (mass) of system elements.

According to [2, 57], the function $\Lambda_{k/l/n}$ for damage interactions in the MTD system is determined by the parameters ρ of the effective energy ratio:

$$\Lambda_{n\backslash k\backslash l} = \Lambda_{n\backslash k\backslash l}\left(\rho_{M\backslash T}, \rho_{n\backslash\tau}\right) \gtrless 1, \tag{69}$$

$$\rho_{n\backslash\tau} = u_\tau^{eff} / u_n^{eff}\,, \quad \rho_{M\backslash T} = u_M^{eff} / u_T^{eff}\,. \tag{70}$$

The quantities Λ calculated by (69) describe the influence of the load parameter ratio on the character and direction of interaction of irreversible damages [2–4]. *If* $\Lambda > 1$, then *the system is self-softening* because at the balance of hardening-softening phenomena, softening processes are dominant. *If* $\Lambda < 1$, *the system is self-hardening*, because at the balance of hardening-softening phenomena, hardening processes are dominant. At $\Lambda = 1$, *the system is stable – the spontaneous hardening-softening phenomena are at balance in it.* The general analysis of damage interactions in the MTD systems will be given on account of its fundamental importance in a separate paper.

After criterion (27) has been basically formalized, the action of electrochemical loads (damages) should be taken

into consideration according to Item VII. It should be said at once that it is difficult to do in the strict mechanothermodynamical statement: electrochemical reactions are very diverse and complex, when the environment interacts with a deformable solid body, and are insufficiently studied. That's why, the approach proposed in [2, 3] is adopted: the simplification is introduced, according to which the damage of solids in the environment is determined by corrosion-electrochemical processes. In addition, *the hypothesis is put forward, following to which the effective energy of corrosion-electrochemical damage is proportional to the square of the corrosion speed*, i.e.

$$u_{ch}^{eff} \sim v_{ch}^2. \tag{71}$$

If according to Item VII, $0 \le D_{ch} \le 1$ is the parameter of corrosion-electrochemical damage of the body, then based on [2, 4, 57], criterion (26) with regard to its shape will be as follows:

$$\Lambda_{M\setminus T}\left[\left(\begin{array}{l}\dfrac{u_n^{eff}\left(\sigma_n^{(V,W)},\,\varepsilon_n^{(V,W)}\right)}{u_0\left(1-D_n\right)}+\\ +\dfrac{u_\tau^{eff}\left(\sigma_\tau^{(V,W)},\,\varepsilon_\tau^{(V,W)}\right)}{u_0\left(1-D_\tau\right)}\end{array}\right)\Lambda_{n\setminus\tau}+\right.$$
$$\left.+\frac{u_T^{eff}}{u_0\left(1-D_T\right)}\right]=1,\ \Lambda \gtrless 1 \tag{72}$$

where

$$0 \le \frac{u_n^{eff}\left(\sigma_n^{(V,W)},\,\varepsilon_n^{(V,W)}\right)}{u_0\left(1-D_n\right)} = \psi_{n(ch)}^{eff} \le 1, \tag{73}$$

$$0 \le \frac{u_\tau^{eff}\left(\sigma_\tau^{(V,W)},\,\varepsilon_\tau^{(V,W)}\right)}{u_0\left(1-D_\tau\right)} = \psi_{\tau(ch)}^{eff} \le 1, \tag{74}$$

$$0 \le \frac{u_T^{eff}}{u_0\left(1-D_T\right)} = \psi_{T(ch)}^{eff} \le 1, \tag{75}$$

$$\begin{aligned} 1-D_T &= b_{e(T)}\left(\frac{v_{ch}}{v_{ch(T)}}\right)^{m_{V(T)}};\\ 1-D_n &= b_{e(n)}\left(\frac{v_{ch}}{v_{ch(n)}}\right)^{m_{V(n)}}; \end{aligned} \tag{76}$$

where v_{ch} is the corrosion speed in this environment, $v_{ch(T)}$, $v_{ch(\sigma)}$, $v_{ch(\tau)}$ is the corrosion speed in the same environment under thermal, force, and friction actions, respectively; b_e are the coefficients responsible for corrosive erosion processes; $m_{V(\bullet)}$ are the parameters responsible for the electrochemical activity of materials at force (the subscript σ), friction (the subscript τ), and thermodynamic (the subscript T) loads, wherein $m_{V(\bullet)} = 2/A_{ch}$ and the parameter $A_{ch} \gtrless 1$.

In [59], other methods for assessment of the parameter D_{ch} .can be found.

As seen, equation (72) is the specification of criterion (27). *According to this criterion, the limiting state of the MTD system is reached when the sum of dialectically interacting irreversible damages at force, friction, and thermodynamic loads (including electrochemical damage when acted upon by stress, friction, temperature) becomes equal to unity*.

Further, consider the particular case when in (46) it is assumed that $A_\sigma(V) = A_\sigma = \text{const}$, $A_\tau(V) = A_\tau = \text{const}$, $A_T(V) = A_T = \text{const}$, $A_{\tau\setminus\sigma}(V) = A_{\tau\setminus\sigma} = \text{const}$, $A_{M\setminus T}(V) = A_{M\setminus T} = \text{const}$.

Firstly, the stress state is caused by volume deformation, for which all components of the stress tensor, except for one component σ (one-dimensional tension-compression, pure bending), can be neglected. Secondly, the stress state is caused by surface friction, for which all components of the stress tensor, except for one component τ_w, can be neglected. Then (40) assumes the following form:

$$\Lambda_{M\setminus T}\left[\Lambda_{\tau\setminus n}\left(A_\sigma\sigma^2 + A_\tau\tau^2\right) + A_T T_\Sigma\right] = u_0,$$

or in accordance with (72)

$$\Lambda_{M\setminus T}\left[\begin{array}{l}\dfrac{a_T}{1-D_T}T_\Sigma+\\ +\Lambda_{\sigma\setminus\tau}\left(\dfrac{a_\sigma}{1-D_\sigma}\sigma^2+\dfrac{a_\tau}{1-D_\tau}\tau_w^2\right)\end{array}\right] = u_0,\ \Lambda \gtrless 1 \tag{77}$$

where $\dfrac{a_\sigma}{1-D_\sigma} = A_\sigma$, $\dfrac{a_\tau}{1-D_\tau} = A_\tau$, $\dfrac{a_T}{1-D_T} = A_n$.

Thus, equation (77) is the simplest form of the energy criterion of the limiting state that is nevertheless of great practical importance [2].

If there is no electrochemical influence of the environment ($D_{ch} = 0$), then

$$u_\Sigma^{eff} = \Lambda_{M\setminus T}\left[a_T T_\Sigma + \Lambda_{\tau\setminus n}\left(a_\sigma\sigma^2 + a_\tau\tau_w^2\right)\right] = u_0,\quad \Lambda \gtrless 1. \tag{78}$$

Equation (78) is the simplest form of the energy criterion of the limiting state, which, nevertheless, is of great practical importance [2, 57, 59]. It serves particularly for the development of methods of assessing the parameters a_T, a_σ, a_τ. In fact, at $\Lambda_{M\setminus T} = \Lambda_{\tau\setminus n} = 1$, the boundary conditions are the following:

$$\left.\begin{array}{ll} T_\Sigma = 0, \tau_w = 0: & a_n\sigma_d^2 = u_0,\quad a_n = u_0/\sigma_d^2;\\ T_\Sigma = 0, \sigma = 0: & a_\tau\tau_d^2 = u_0,\quad a_\tau = u_0/\tau_d^2;\\ \sigma = 0, \tau_w = 0: & a_T\sigma_d = u_0,\quad a_T = u_0/T_d, \end{array}\right\}, \tag{79}$$

where σ_d, τ_d are the force and friction limiting stresses as $T \to 0$. These are called the limits of (mechanical) destruction, T_d is the destruction temperature (when $\sigma = 0$, $\tau_w = 0$) or the thermal destruction limit.

The effective ("dangerous") part of total energy of strain can also be determined from the following physical considerations. It shall be assumed that the strain energy flux u generated in the material sample during its cyclic strain ($\varepsilon = \varepsilon_{\max}\sin\omega t$) in the homogeneous (linear) stress state is to a certain extent *similar to the light flux*. In fact, it is continuously excited when the loading cycle is repeated with the speed $\omega = 1/\lambda$. This enables one to consider it as a wave (with the length λ). Some part of the energy u generated in such a way can be absorbed by material atoms and structural formations, followed by damage of material. Denote the absorbed part of the energy by u^{eff}. Then the generated energy u is equal to:

$$u = u^{eff} + u_{cons}, \tag{80}$$

where u_{cons} is the non-absorbed part (it is called the conservative part) of the generated energy u.

If the analogy of light and energy strain is justified, then the strain absorption law may be similar to Bouguer's light absorption law. Consequently, the equation relating the energy u_{cons} passed through the material strain volume V and the generated energy u is of the form:

$$u_{cons} = u\exp(-\chi_\varepsilon V), \tag{81}$$

or, in accordance with *Lambert*, in differential form:

$$\frac{du}{u} = -\chi_\varepsilon V\ . \tag{82}$$

Here as in *Bourguer – Lambert's* equation, the coefficient χ_ε independent of u is the energy absorption parameter.

Taking into account (81) into (80), *the strain energy absorption law* is obtained:

$$u^{eff} = u\left[1-\exp(-\chi_\varepsilon V)\right], \tag{83}$$

and hence if $u = 0$ or $V = 0$ then $u^{eff} = 0$. If $V \to \infty$ it appears that according to (81) $u_{cons} = u$, i.e., all input energy is dissipated within such a volume.

From the physical point of view, the strain energy absorption process is caused by many phenomena:

- transition of electrons in absorbing atoms from lower to higher energy levels (quantum theory);
- generation and development of dislocation structures (dislocation theory);
- emergence of II and III order residual strains (stresses) (elasticity theory);
- formation and development of any imperfections (defects) of material composition and structure – point, planar and spatial (physical materials science);
- hardening-softening phenomena (including strain aging) developing in time (fatigue theory);
- changes in (internal) Tribo-Fatigue entropy (wear-fatigue damage mechanics [2]).

It should be noted that approach (83) can also be extended to the case of friction, since any indenter drives a strain wave upstream in the thin surface layer of the solid to which indenter is pressed to. Energy absorption parameter in this case will be χ_γ where the subscript γ denotes the shear strain. Similarly, heat absorption in the deformable solid body can also be considered. Finally, the problem of strain energy absorption in the non-uniform (including complex) stress state can be easily solved by putting the dangerous volume $V = V_{P_\gamma}$ into (81)–(83).

It should be noted that *although criterion (78) is special, it is fundamental and general in nature.* Its general nature is caused by the fact that in this case, all four particular phenomena responsible for the MTD system state (in the statement simplified in terms of the stress-strain state) are taken into account (in accordance with Item III). Its fundamental nature is that here, as in complete solution (46), $\Lambda_{n\setminus\tau}$ takes into account the interaction of effective mechanical energy components caused by friction τ_w and normal σ stresses, whereas $\Lambda_{M\setminus T}$ takes into account the interaction of the thermal and mechanical components of the effective energy. The effective energy thermal component is determined by the variations of the total temperature $T_\Sigma = T_2 - T_1$ in the bodies contact zone caused by all heat sources, including the heat released during mechanical (spatial and surface) strain, structural changes, etc.

Mechanothermodynamical states. Within the framework of mechanothermodynamics a special approach is being developed to assess the entropy in terms of a generalized energy state. Following this approach and formula (77), out of the *total energy* (specific) due to some particular loads (force, temperature, etc.), *its effective part directly spent for the damage production is defined* by the experimentally found coefficients A_l in formulas (41), (42), (77) [2–32]

$$u_l^{eff} = A_l u_l, \tag{84}$$

where the u_l are the specific internal energies at tear (u_n), shear (u_τ), thermal action (u_T).

The *total specific energy of an elementary volume* and a *rate of its change* are then given as

$$u = \sum_l \left[(1-A_l)u_l + u_l^{eff}\right]; \tag{85}$$

$$\frac{du}{dt} = \sum_l \left[(1-A_l)\frac{du_l}{dt} + \frac{du_l^{eff}}{dt} \right]. \tag{86}$$

Moreover, the Λ-functions are used to take into consideration a complex (nonadditive) character of interactions between effective energies of different nature expressed by formula (42). This allows the *total effective energy of the system* to be assessed:

$$\begin{aligned} u_\Sigma^{eff} &= \Lambda_\alpha\left(u_l^{eff}\right) = \Lambda_{M\setminus T}\left(\Lambda_{\tau\setminus n}, A_l u_l\right) = \\ &= \Lambda_{M\setminus T}\left\{\Lambda_{\tau\setminus n}\left[A_n u_n + A_\tau u_\tau\right] + A_T u_T\right\}, \end{aligned} \tag{87}$$

where the Λ_α are the possible combinations of interaction of effective energies (irreversible damages).

The *specific feature of* Λ-functions is such that

$$u_\Sigma^{eff} \gtrdot u_l^{eff}, \tag{88}$$

and, hence,

$$u_\Sigma^{eff} \gtrdot \sum u\ . \tag{89}$$

Thus, using coefficients A_l and Λ-functions it is possible to assess *energy interaction* due to different-nature loads. Such interaction can cause both a sharp growth and a substantial decrease of effective energy, resulting in damages and limiting states, as compared to the one calculated by the ordinary additiity model of type (17):

$$u_\Sigma = \sum A_l u_l. \tag{90}$$

The *total effective energy* of volume V and its accumulation in time with regard to formula (87) are of the form

$$u_\Sigma^{eff} = \int_V \rho u_\Sigma^{eff}(V)dV \tag{91}$$

and

$$u_\Sigma^{eff}(t) = \int_t \int_V \rho u_\Sigma^{eff}(V,t)dVdt. \tag{92}$$

The principal moment of the mechanothermodynamical model is the *account of the limiting state* (limits of plasticity, strength, fatigue, etc.) according to Item XIII (see section 3)

$$u_{\Sigma}^{eff} = u_0, \tag{93}$$

where u_0 is the limiting density of the internal energy interpreted as the initial activation energy of the disintegration process.

A relationship between the current state (mechanical, thermomechanical, energy) of an elementary volume of a solid (medium) and its limiting state enables one to construct the *parameter of local energy damageability*: dimensionless

$$\psi_u^{eff} = \frac{u_{\Sigma}^{eff}}{u_0} \tag{94}$$

or dimensional

$$\psi_{u^*}^{eff} = u_{\Sigma}^{eff} - u_0. \tag{95}$$

Local energy damageability (94) or (95) is most general out of the damageability parameters constructed in terms of different mechanical (thermomechanical) states φ [2–32]:

$$\psi_q = \varphi_q / \varphi_q^{(*\lim)}, \tag{96}$$

where φ = σ, ε, *u*; the σ are the stresses; the ε are the strains; *u* is the density of internal energy; the $\varphi_q^{(*\lim)}$ are the limiting values of the state φ $q \in \{eqv, ij, i, S, {}_{ij}^{D}, n, \tau, \text{int}, u, {}_{u}^{n}, {}_{u}^{\tau}, {}_{u}^{eff}\}$; *eqv* is the equivalent mechanical state; the *ij* are the components of the tensor φ; the *i* are the main components of the tensor φ; *S* and ${}_{ij}^{D}$ are the sphere and deviator parts of the tensor φ; *n* and τ are the normal and tangential components of the tensor φ; int is the intensity φ; *u* is the specific potential strain energy (internal energy density); the indices at *u* mean: ${}_{u}^{n}$ and ${}_{u}^{\tau}$ are the specific potential strain energy at tension-compression and shear; ${}_{u}^{eff}$ is the effective specific potential strain energy.

Integral damageability measures can be built on the basis of local measures (96) *with the use of the model of a deformable solid with a dangerous volume* (64)–(68) [53].

By the *dangerous volume* is understood the spatial region of a loaded solid, at each point of which the value of local damageability is smaller than the limiting one [2–32]:

$$V_q = \left\{ dV / \varphi_q \geq \varphi_q^{(*\lim)}, dV \subset V_k \right\}, \tag{97}$$

or

$$V_q = \left\{ dV / \psi_q \geq 1, dV \subset V_k \right\}.$$

Dangerous volumes are calculated by the following general formula:

$$V_q = \iiint\limits_{\psi_q(V) \geq 1} dV. \tag{98}$$

The *integral condition of damageability* of a solid or a system can be written in the form

$$0 < \omega_q = \frac{V_q}{V_0} < 1, \tag{99}$$

where V_0 is the working volume of the solid.

To analyze at a time dangerous volumes local damageability distributed within them, the *function of damageability of unit volume* is introduced

$$d\Psi_q = \psi_q(V) dV. \tag{100}$$

The *function of damageability of the entire volume V* will then be as

$$\Psi_q = \int\limits_{\psi_q \geq 1} \psi_q(V) dV. \tag{101}$$

The simplest functions of *damageability accumulation in time* for unit volume and the entire volume will be of the following form, respectively

$$d\Psi_q^{(t)} = \int\limits_t \psi_q(t) dt; \tag{102}$$

$$\Psi_q^{(t)} = \int\limits_{\psi_q \geq 1} \int\limits_t \psi_q(V,t) dt dV. \tag{103}$$

The *indices of volume-mean damageability*

$$\overline{\Psi}_q^{(V)} = \frac{1}{V_q} \int\limits_{\psi_q \geq 1} \psi_q(V) dV \tag{104}$$

and *its accumulation in time* can be used

$$\overline{\Psi}_q^{(V,t)} = \frac{1}{V_q} \int\limits_t \int\limits_{\psi_q \geq 1} \psi_q(V,t) dV dt. \tag{105}$$

The analysis of formulas (94), (100), (102) comes to the conclusion that conceptually, they are related to the concept of entropy as a difference (or relations) between two states (configurations) of a system, the degree of its organization (chaotic state). As applied to damageability, such states are current and limiting.

Now using local energy damageability (94), construct *specific* (per unit mass) *Tribo-Fatigue entropy* (up to a constant):

$$s_{TF} = \psi_u^{eff}(\Lambda_\alpha, A_l, \sigma_{ij}, T) = \lim_{\Delta m \to 0} A_\psi \frac{u_{\Sigma}^{eff}(\Delta m)}{u_0 \Delta m}, \tag{106}$$

or

$$s_{TF} = s_{TF^*} = \frac{\psi_{u^*}^{eff}(\Lambda_\alpha, A_l, \sigma_{ij}, T)}{T} = \frac{u_{\Sigma}^{eff} - u_0}{T}. \tag{107}$$

where A_ψ is the dimensional parameter (J·mol^{-1}·K^{-1}).

On the basis of the expressions for entropy (18), as well as of formulas (85), (86) the *local entropy* and the *rate of its change within an elementary volume* will be

$$s = \frac{1}{T} \sum_l \left[(1 - A_l) u_l \right] + s_{TF} \tag{108}$$

and

$$\frac{ds}{dt} = \frac{1}{T} \sum_l \left[(1 - A_l) \frac{du_l}{dt} \right] + \frac{ds_{TF}}{dt}. \tag{109}$$

From formulas (108) and (109), it is seen that unlike the thermomechanical model, the *state indices of the mechanothermodynamical system u* and *s are not*

equivalent. This is due to the fact that the calculation of the Tribo-Fatigue entropy s_{TF} by formula (106) is supplemented by the limiting state in the form of the limiting density of the internal energy u_0.

The Tribo-Fatigue entropy S_{TF} is calculated not within the entire volume V, but only within its damageable part, i.e., within the energy effective dangerous volume V_u^{eff}:

$$V_u^{eff} = \left\{ dV / u_\Sigma^{eff} \geq u_0, dV \subset V_k \right\} . \qquad (110)$$

On the basis of formulas (11), (106) and (110), the *Tribo-Fatigue entropy of volume V* will be

$$S_{TF} = \int\limits_{u_\Sigma^{eff}(V) \geq u_0} \rho s_{TF}(V) dV = \int\limits_{u_\Sigma^{eff}(V) \geq u_0} \rho \psi_u^{eff}(V) dV, \qquad (111)$$

where

$$\psi_u^{eff}(V) = \frac{u_\Sigma^{eff}(V)}{u_0}$$

or

$$\psi_u^{eff}(V) = \frac{\psi_{u^*}^{eff}(V)}{T} = \frac{u_\Sigma^{eff}(V) - u_0}{T(V)}, \qquad (112)$$

and *its accumulation* will be

$$S_{TF}(t) = \int\limits_t \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho s_{TF}(V,t) dV dt = \int\limits_t \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho \psi_u^{eff}(V,t) dV dt, \qquad (113)$$

where

$$\psi_u^{eff}(V,t) = \frac{u_\Sigma^{eff}(V,t)}{u_0}$$

or

$$\psi_u^{eff}(V,t) = \frac{\psi_{u^*}^{eff}(V,t)}{T(V,t)} = \frac{u_\Sigma^{eff}(V,t) - u_0}{T(V,t)} . \qquad (114)$$

The *principal feaure of Tribo-Fatigue total* S_{TF} and specific s_{TF} entropies should be emphasized. They allow the difference between two states to be assesses not only quantitatively (as thermomechanical entropy), but also qualitatively, as the value of the limiting density of the internal energy u_0 is explicitly introduced into the calculation of the specific entropy s_{TF}. Thus, s_{TF} and S_{TF} allow one to answer the question how much the current state of a solid or a system is dangerous in comparison with limiting states.

The *total entropy* and the *rate of its change for a solid of a system* with regard to (111) and (113) assume the form

$$S = \int\limits_V \frac{1}{T(V)} \sum_l \rho \left[\left(1 - A_l(V)\right) u_l(V) \right] dV + S_{TF} \qquad (115)$$

and

$$\frac{dS}{dt} = \int\limits_V \frac{1}{T(V)} \sum_l \rho \left[\left(1 - A_l(V)\right) \frac{du_l(V)}{dt} \right] dV + \frac{dS_{TF}}{dt} . \qquad (116)$$

Based on formulas (106)–(116), the *function of accumulation of total entropy in time* can be built*:*

$$S(t) = \int\limits_t \int\limits_V \sum_l \rho s_l(V,t) dV dt + \int\limits_t \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho s_{TF}(V,t) dV dt = \int\limits_t \int\limits_V \frac{1}{T(V,t)} \sum_l \rho \left[\left(1 - A_l(V,t)\right) \frac{du_l(V,t)}{dt} \right] dV dt + \int\limits_t \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho \psi_u^{eff}(V,t) dV dt. \qquad (117)$$

In this respect, bearing in mind the limiting states of a solid or a system, models (115)–(117) permit one to answer the question whether the current state is the point of a qualitative jump in the system, i.e., whether the current state is close to the limiting one. A similar (*dialectical as a matter of fact*) *qualitative transition* differs, for example, from the bifurcation point having the uncertainty in a further development of events and the possibility to predict the system behavior after a transition on the basis of the analysis of s_{TF} and S_{TF}. Particular limiting states (limit of strength, mechanical or contact fatigue, etc.) enable one to predict the situation after transition the given point: principal changes in the system properties and behavior or the formation of a new system based on the previous one.

As an example, there can be non-linear deformation or generation of microcracks in the solid (or the system) that cause the changes in its strength and fatigue properties, and, hence, to its response to loads. In turn, formed macrocracks lead to local continuum violation – formation of new free surfaces (possibly, of new solids – destruction products), i.e. a new system.

It should be noted that models (115)–(117) were built using a traditional concept of entropy additivity (10) although with regard to substational improvements. These models also contain reversible processes described by the entropy components s_l not yielding primary damages and, hence, the limiting states – the points of a qualitative change of the system.

It is therefore more advisable for a qualitative and quantitative analysis of evolution of systems (whose states are tradiationally defined as bifurcation branches) that the entropy state should be determined using the mechanothermodynamical model of the solid using only Tribo-Fatigue entropy. In this case, formulas (111)–(113) for entropy and its accumulation will be of the form

$$S = S_{TF} = \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho s_{TF}(V) dV = \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho \psi_u^{eff}(V) dV , \qquad (118)$$

and

$$S(t) = S_{TF}(t) = \int\limits_t \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho s_{TF}(V) dV dt = \int\limits_t \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho \psi_u^{eff}(V,t) dV dt. \qquad (119)$$

To identify the points of qualitative change in the limiting states of solids (systems), the indices of relative integral entropy and its accumulation built on the basis of the concept of integral condition of damageability of a solid (99) can be used:

$$\omega_S = \frac{S_{TF}}{V_0} = \frac{1}{V_0} \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho s_{TF}(V) dV; \qquad (120)$$

$$\omega_S(t) = \frac{S_{TF}(t)}{V_0} = \frac{1}{V_0} \int\limits_t \int\limits_{u_\Sigma^{eff}(V,t) \geq u_0} \rho s_{TF}(V) dV dt. \qquad (121)$$

The indices S_{TF}, $S_{TF}(t)$, ω_S, $\omega_S(t)$ *can grow infinitely, allowing not only the limiting states of type (93), but also different transmitting states to be described;* in essence, they "provide" a quantitative description of the *law of increase of entropy*.

Now based on formulas (24), (115), (117) and (119) let us construct *generalized expressions for entropy, a rate of its change* as well as *its accumulation in the mechanothermodynamical system consisting of a liquid (gas) medium of volume V* and *a solid of volume* V_ψ:

$$S = \int_V \rho s_T dV + \int_{V_\psi} \sum_l \rho s_l dV_\psi + \int\limits_{u_\Sigma^{eff} \ge u_0} \rho s_{TF} dV_\psi =$$
$$= \int_V \frac{1}{T} \sigma_{ij} \varepsilon_{ij} dV + \int_V \frac{1}{T} \rho q dV + \int_V \frac{1}{T} \rho \sum_k \mu_k n_k \; dV +$$
$$+ \int_{V_\psi} \frac{1}{T} \sum_k \rho \left[(1 - a_k) u_k \right] dV_\psi + \int\limits_{u_\Sigma^{eff} \ge u_0} \rho \psi_u^{eff} dV_\psi; \qquad (122)$$

$$\frac{dS}{dt} = \int_V \rho \frac{ds_T}{dt} dV + \int_{V_\psi} \sum_l \rho \frac{ds_l}{dt} dV_\psi + \int_{V_\psi} \rho \frac{ds_{TF}}{dt} dV_\psi =$$
$$= \int_V \frac{1}{T} \sigma_{ij} \frac{d\varepsilon_{ij}}{dt} dV + \int_V \frac{1}{T} \rho \frac{dq}{dt} dV + \int_V \frac{1}{T} \rho \sum_k \mu_k \frac{dn_k}{dt} \; dV +$$
$$+ \int_{V_\psi} \frac{1}{T} \sum_k \rho \left[(1 - a_k) \frac{du_k}{dt} \right] dV_\psi + \int\limits_{u_\Sigma^{eff} \ge u_0} \rho \frac{d\psi_u^{eff}}{dt} dV_\psi; \qquad (123)$$

$$S(t) = \int_t \int_V \rho s_T dV dt + \int_t \int_{V_\psi} \sum_l \rho s_l dV_\psi dt +$$
$$+ \int_t \int\limits_{u_\Sigma^{eff} \ge u_0} \rho s_{TF} dV_\psi dt = \int_t \int_V \frac{1}{T} \sigma_{ij} \varepsilon_{ij} dV dt +$$
$$+ \int_t \int_V \frac{1}{T} \rho q dV dt + \int_t \int_V \frac{1}{T} \rho \sum_k \mu_k n_k \; dV dt +$$
$$+ \int_t \int_{V_\psi} \frac{1}{T} \sum_l \rho \left[(1 - a_l) u_l \right] dV_\psi dt +$$
$$+ \int_t \int\limits_{u_\Sigma^{eff} \ge u_0} \rho \psi_u^{eff} dV_\psi dt. \qquad (124)$$

Entropy state indices can be built similarly for a system composed of many media.

It should be noted that the interaction (contact) of two media in formulas (122)–(125), which can be complex in nature, is taken into account only implicitly in terms of medium state parameters (stresses, strains, temperature). It is obvious that this is only the first step to a comprehensive (generalized) solution of the problem stated.

The simplified writing of expression (123) for the entropy increment of the mechanothermodynamical system composed of finite volumes dV and dV_ψ given in [32] can be re-written in the following form:

$$dS = (dS)_T + (d_i S)_{TF} = \frac{du + \Delta p dV}{T} -$$
$$- \frac{1}{T} \sum_k \mu_k dN_k + \Psi_u^{eff} dV_\psi. \qquad (125)$$

Expression (125) can also be represented in terms of specific quantities as:

$$dS = \int_V \frac{\rho du + \rho dp}{T} dV - \int_V \frac{1}{T} \rho \sum_k \mu_k dn_k \; dV +$$
$$+ \int\limits_{u_\Sigma^{eff} \ge u_0} \rho d\psi_u^{eff} dV_\psi \qquad (126)$$

or on the basis of (123) as:

$$\frac{dS}{dt} = \int_V \frac{\sigma_{ij} d\varepsilon_{ij} + \rho dq}{T dt} dV - \int_V \frac{1}{T} \rho \sum_k \mu_k \frac{dn_k}{dt} \; dV +$$
$$+ \int\limits_{u_\Sigma^{eff} \ge u_0} \rho \frac{d\psi_u^{eff}}{dt} dV_\psi. \qquad (127)$$

In formulas (111)–(113) for calculation of The Tribo-Fatigue entropy S_{TF} and its accumulation $S_{TF}(t)$, the specific entropy s_{TF} is assumed to be integrated in terms only of the damageable part of the solid – the dangerous volume. However the influence of undamagable regions can also be allowed for by integrating S_{TF} over the entire volume:

$$S_{TF} = \int_V \rho s_{TF}(V) dV = \int_V \rho \psi_u^{eff}(V) dV; \qquad (128)$$

$$S_{TF}(t) = \int_t \int_V \rho s_{TF}(V,t) dV dt =$$
$$= \int_t \int_V \rho \psi_u^{eff}(V,t) dV dt, \qquad (129)$$

where

$$\psi_u^{eff} = \begin{cases} \dfrac{u_\Sigma^{eff}(V,t)}{u_0} \ge 1, \text{ if } u_\Sigma^{eff} \ge u_0; \\ \dfrac{u_\Sigma^{eff}(V,t)}{u_0} < 1, \text{ if } u_\Sigma^{eff} < u_0, \end{cases} \qquad (130)$$

or

$$\psi_u^{eff} = \frac{\psi_{u*}^{eff}(V,t)}{T(V,t)} = \begin{cases} \dfrac{u_\Sigma^{eff}(V,t) - u_0}{T(V,t)} \ge 0, \text{ if } u_\Sigma^{eff} \ge u_0; \\ \dfrac{u_\Sigma^{eff}(V,t) - u_0}{T(V,t)} < 0, \text{ if } u_\Sigma^{eff} < u_0. \end{cases} \qquad (131)$$

From expression (131), it is seen that $\psi_u^{eff} < 0$ is observed outside the dangerous volume (at $u_\Sigma^{eff} < u_0$). This means that the *specific Tribo-Fatigue entropy* s_{TF} *also appears to be negative* (or less than unity for its alternative definition) *outside the dangerous volume* where the limiting state is not reached. Negative values of ψ_u^{eff} and s_{TF} can then be interpreted as the absence of damageability or in other words as the retention of structure and/or properties of the solid.

As follows from the above-stated, the assumption on the entropy additivity is wrong in the general case for a system composed of both a solid and a liquid (gas) where chemical reactions can occur. By analogy with Λ-functions, interaction functions of different-nature energy (179) it is necessary to introduce *interaction functions of different-nature entropy* by adding them to expression (125) in effort to determine total effective entropy:

$$dS_{total}^{eff} = \Lambda_{T\backslash TF}^{(S)}\left(dS_T + d_i S_{TF}\right) = \Lambda_{T\backslash TF}^{(S)}\left[\Lambda_{Q\backslash Ch}^{(S)}\left(dS_T^Q + dS_{Ch}^Q\right) + d_i S_{TF}\right] = \Lambda_{T\backslash TF}^{(S)}\left[\begin{array}{l}\Lambda_{Q\backslash Ch}^{(S)}\left(\dfrac{du+\Delta pdV}{T} - \dfrac{1}{T}\sum_k \mu_k dN_k\right) + \\ + \Psi_u^{eff} dV_\psi\end{array}\right], \quad (132)$$

or

$$dS_{total}^{eff} = \Lambda_{T\backslash TF\backslash Ch}^{(S)}\left(dS_T + d_i S_{TF}\right) = \Lambda_{T\backslash TF\backslash Ch}^{(S)}\left[\begin{array}{l}\dfrac{du+\Delta pdV}{T} - \\ -\dfrac{1}{T}\sum_k \mu_k dN_k + \Psi_u^{eff} dV_\psi\end{array}\right], \quad (133)$$

where the subscripts *Q* and *Ch* denote the thermodynamic and chemical entropy components.

Formulas (132)–(133) are supplemented by the *generalized interaction functions* $\Lambda_{T\backslash TF}^{(S)}$, $\Lambda_{Q\backslash Ch}^{(S)}$, $\Lambda_{T\backslash TF\backslash Ch}^{(S)}$ *in mechanothermodynamical systems*. This means that the *hypothesis on the additivity of thermodynamic and Tribo-Fatigue entropy is not accepted*. The appropriate Λ-functions of interactions should then be specified and introduced into equations (132)–(133).

Translimiting states. According to the available information, the *theory of translimiting states* is still insufficiently developed [2]. The elements of this theory will be set forth on the basis of solutions (72), (76), and (77).

Figure 4 shows the general analysis of the contribution of *mechanical-chemical-thermal damage* (parameters D) to the process of reaching the limiting state by the MTD system.

Having studied formulas (72), (76) and Figure 4, the following conclusions can be drawn.

1. The growth of parameters D means a decrease in the relative damage speed $v_{ch} / v_{ch(*)}$ (Figure 4, *a*). In other words, mechanical-chemical-thermal damage speeds up the process of reaching the limiting state by the MTD system the faster, the greater is the value of the parameter D and/or the speed $v_{ch(*)}$.

2. The parameter m_v influences greatly the system damage, and this influence is the greater, the larger is this parameter (Figure 4, *b*). The important feature of this influence is that this environment is very sensitive to the excitation of mechanical stresses in the MTD system and to the temperature rise if the parameter $m_v > 5$. In other words, in such a case, the translimiting state can be realized, for which the damage measure (53) is more than unity ($\psi_u^{eff} > 1$), whereas according to (52), it is sufficient to have $\psi_u^{eff} = 1$ to reach the limiting state.

Two specific cases are illustrated in Figure 4, *c*.

The first case – $D = 0$. There is no electrochemical corrosion influence on wear-fatigue damage. But this doesn't mean that the electrochemical corrosion process does not occur. In fact, according to (76) when $D = 0$, we have (if $m_v = 1$):

$$1 - \frac{v_{ch}}{v_{ch(*)}} b_* = 0,$$

this implies that the situation should be the following: $b_* = 1$ and $v_{ch} / v_{ch(*)} = 1$, i.e., the corrosion speed is insensitive to this factor (mechanical or frictional stresses). This means that *threshold values* of σ^0, τ_w^0, and T^0 exist for a given environment. The corrosion speed in such an environment does not vary for $\sigma \le \sigma^0$, $\tau_w \le \tau_w^0$ and $T_\Sigma \le T^0$ (see formula (77)).

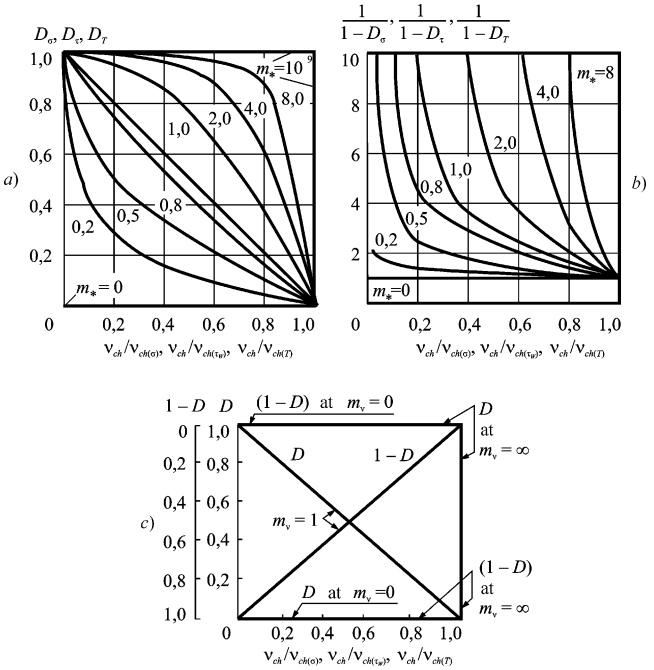


Figure 4 – Analysis of the influence of mechanical-chemical-thermal processes on the system damage

The second case – D = 1 and, hence, $1/(1 - D) \to \infty$. Explosive damage is realized within the system as $\psi_u^{eff} \to \infty$. In this case, it should be

$$\frac{v_{ch}}{v_{ch(*)}} b_* = 0.$$

Since $v_{ch} = 0$ is the impossible event then it can be assumed that $v_{ch(*)} \to \infty$. This is *the condition of mechanical-chemical-thermal explosion* in the MTD system. The explosion is caused not just by the environmental impact – it is the environmental impact greatly increased by temperatures and mechanical stresses.

Thus, complex function (72) for the damage of MTD systems can also be used for *analyzing their translimiting states* caused by a supercritical growth of thermodynamic, mechanical, frictional, and electrochemical loads according to formulas (73)–(76), i.e.,

$$1 \le \psi_u^{eff} = \Lambda_{T\backslash M}\left[\psi_{T(ch)} + \Lambda_{n\backslash\tau}\left(\psi_{n(ch)} + \psi_{\tau(ch)}\right)\right] \le \infty \quad (134)$$

According to (134), there are *many translimiting states* of the MTD system defined by the condition $\psi_u^{eff} > 1$. This is possible in those (many) cases when the state of the system, critical in terms of its damage, is reached not at one but many points of the dangerous volume. Hence the assumption can be made that *many (different) forms* of these states must exist.

Above-mentioned criterion equations (43), (47), (52), (58), (72), and (77) are obtained from the consideration of the energy conditions when the limiting state has been reached. It is stated that they can in principle be used for

describing a variety of translimiting states, but only in those cases when situations in the MTD system are created for *an unconditional supercritical* (essentially unrestrained) growth of loads (explosions, accidents, disasters, fires, etc.).

Another more general approach for the analysis of translimiting states is that it considers *damage space* defined according to (59), (64) by volume measures

$$0 \le \omega_{ij} = \frac{V_{ij}}{V_0} \le 1. \qquad (135)$$

On the basis of (72)–(76) the spatial damage measures can be defined as

$$\omega_{\sigma(ch)} = \frac{V_{P\gamma}}{V_0(1-D_\sigma)};$$

$$\omega_{\tau(ch)} = \frac{S_{P\gamma}}{S_0(1-D_\tau)}; \qquad (136)$$

$$\omega_{T(ch)} = \frac{V_{T\gamma}}{V_0(1-D_T)},$$

where V_0, S_k are the working volumes. So criterion (77) can be written with regard to (136):

$$\Lambda_{T\backslash M}\left[\frac{V_{T\gamma}}{V_0(1-D_T)} + \Lambda_{\sigma\backslash\tau}\left(\frac{V_{P\gamma}}{V_0(1-D_\sigma)} + \frac{S_{P\gamma}}{S_0(1-D_\tau)}\right)\right] = 1. \qquad (137)$$

The advantage of (137) is that the interaction of dangerous volumes [2] at different loads is taken into account when the limiting state of MTD systems is formed. In addition, as mentioned above, since absolute damage volumes are determined by a number of structural-technological and metallurgical factors (59), these factors appear to be automatically accounted for in the limiting state criterion for such systems.

If the rupture of interatomic bonds takes place only over one dangerous section of an object at all "points" of this section $(u_\Sigma^{eff} = u_0)$, then it is divided into two parts, which corresponds to the condition $\omega_\Sigma = 1$. But if the complex of loads (mechanical, electrochemical, thermodynamic, etc.) is such that "all" interatomic bonds undergo rupturing over this section, then the process occurs that is called *disintegration of object* whose death corresponds to the condition $\omega_\Sigma^* = \infty$. This is *the most common form of the translimiting state*: the system disintegrates into an infinite number of particles of arbitrarily small size (for example, atoms). It is clear that there must be some *intermediate forms of the limiting states of the system.* The condition of their implementation is

$$1 \le \omega_\Sigma^* = \Lambda_{T\backslash M}\left[\left(\omega_{\sigma(ch)} + \omega_{\tau(ch)}\right)\Lambda_{\sigma\backslash\tau} + \omega_{T(ch)}\right] \le \infty. \qquad (138)$$

Naturally, equation (138) is similar to (134). Their difference is that conditions (134) are written in terms of energy damage measures, while conditions (138) – in terms of volume (space) damage measures.

The general classification of conceivable *states of object in terms of volume damage* is given in Table 1 that is similar to the one in Table 1 [1], but with the difference that a special index (asterisk *) is introduced for translimiting states.

The probability interpretation of irreversible damage events in the MTD system can be made.

Table 1 – **Characteristics of the states of objects**

A-state	Undamaged	$\omega_\Sigma = 0$	*A*-evolution: characteristic states of a system (damage)
B-state	Damaged	$0 < \omega_\Sigma < 1$	
C-state	Critical (limiting)	$\omega_\Sigma = 1 = \omega_c$	
D-state	Supercritical (translimiting)	$1 < \omega_\Sigma^* < \infty$	
E-state	Disintegration	$\omega_\Sigma^* = \infty$	

If

$$0 \le P(\omega_\Sigma) \le 1 \qquad (139)$$

is *the classical probability of the MTD system failure in terms of damage* $(0 \le \psi_\Sigma \le 1)$ within the time interval $(t_0, T_\oplus)$ (item XIV) then $P(\omega_\Sigma = \omega_c = 1) = 1$ is *the reliable probability of unconditional functional failure*. For supercritical states *the concept of reliable probability* for supercritical damages [60] is introduced

$$1 < P_*\left(\omega_\Sigma^*\right) \le \infty. \qquad (140)$$

Supercritical damages $(1 < \omega_\Sigma^* < \infty)$ correspond to numerous and infinite shapes and sizes of particles that are formed in the process of degradation (disintegration) of the system.

Figure 5 illustrates the relationship between the system damage system and probability.

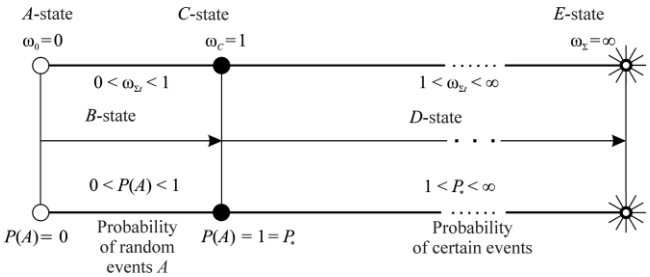


Figure 5 – Relationship between the system damage and probability

Note that the data in Table 1 can be interpreted in the following way. If

$$\omega_\Sigma^* \to \infty, \qquad (141)$$

then the absolute size of forming particles should be as small as desired according to (32), i.e.,

$$d_\omega^* \to 0. \qquad (142)$$

Assume, to a first approximation, the logarithmic relationship between d_ω and ω_Σ. Then

$$d_{\omega}^{*} = e^{-\omega_{\Sigma}^{*}} \quad \text{or} \quad \omega_{\Sigma}^{*} = -\ln d_{\omega}^{*} \tag{143}$$

As it follows from the above-mentioned, all states of the MTD system are predicted by the corresponding equations (134) and/or (138). A drawback of this prediction or description is that the dependence of damage measures [for example (134)] on the determining parameters appears to be smooth over the entire range $0 \le \omega_{\Sigma} \le \infty$ (Figure 6, *a*). However it should be noted that this is valid only in the case (essentially, in the ideal case) when the values of the determining parameters (σ, τ_w, Λ, etc.) are continuously increasing. But the *surface of damage has jumps* (*discontinuities*) whenever the jumps of any load or any abrupt changes in hardening-softening processes (Figure 6, *b*, *c*) are realized. It is easy to understand that in reality these specific situations lead to damage jumps, i.e. to *qualitative changes* or *system state transformations*. It should be added that our approach has a special advantage: it is based on the analysis of *damage as a physical reality* independent of the fact what damage mechanisms are already known and what mechanisms will be clarified.

The last remark is of particular importance. The fact is that when there is *"conventional mechanical fracture of a regular mechanical object"* ($\omega_{\Sigma} = 1$), i.e., it disintegrates, at least, into two parts, the existence of the MTD system does not end – in accordance with Item XIV, a long period comes when the object disintegrates into particles ($1 < \omega_{\Sigma}^{*} \le \infty$). Here not so much mechanical loads, as electrochemical and thermodynamic phenomena (processes) are the determining parameters. On the basis of the above-said, the *law of disintegration of the MTD system* is formulated in the form

$$\sum m_{V_{ijT}} = m_{V_0}. \tag{144}$$

Law (144) suggests the mass conservation of the system regardless of the conditions of its degradation and disintegration. In other words, the *mass of disintegrating particles* $\sum m_{V_{ijT}}$ (whatever their size) *cannot exceed* (or it can be less than) *the initial mass* m_{V_0} of the *MTD system*.

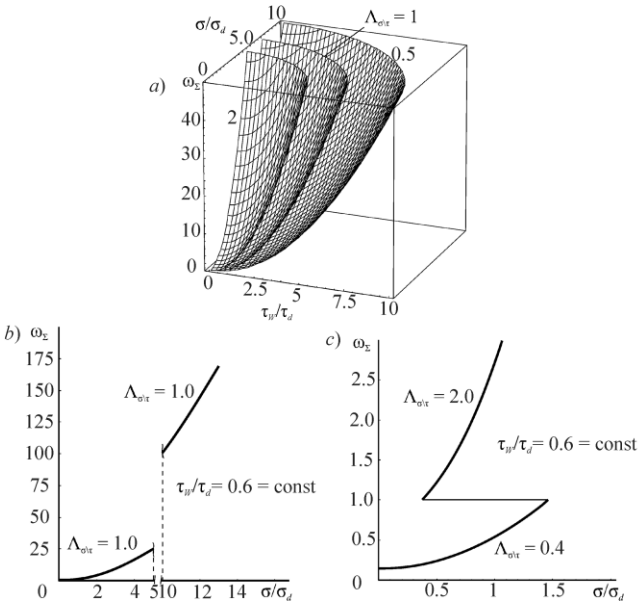
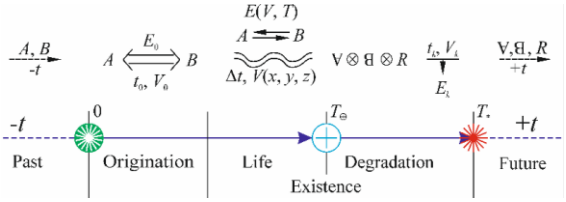


Figure 6 – Formation of damage surfaces (*a*) and functions (*b*, *c*) ω_{Σ} due to the changes in the determining parameters ($(\sigma / \sigma_d > 0, \tau / \tau_d > 0, \Lambda_{\sigma\backslash\tau} > 0)$

Hence, there is the need of the analysis (at least, short) of the evolution of systems

Evolution of the MTD system. Give the description [61] of the behavior of the deformable *solid – solid* system in some environment on the most general – *dialectical grounds*. The origination of the system (*occurrence*, *life* and *degradation*) can be represented in the following general form (145):



(145)

Here *A*, *B* are some separate bodies (elements, etc.), details, objects. Their existence in the past (–t) is sketched by the dashed line with the arrow.

This writing

$$A \overset{E_0}{\Longleftrightarrow} B \qquad t_0,\ V_0 \tag{146}$$

means that the creation (origination) of a system (or an object) is the product of energy (E_0) interaction ($\leftrightarrow$) of bodies *A*, *B* implemented in the time t_0 within the volume V_0. Of course, this product is neither *A* nor *B*; but it is an entity with special integral characteristics and functions which neither *A* nor *B* can possess.

The writing

$$\begin{array}{c} E(V,\ T) \\ A \rightleftarrows B \\ \sim\sim\sim \\ \Delta t,\ V(x,\ y,\ z) \end{array} \tag{147}$$

means that the life of the system is the process of its energy $E(V, t)$ interaction with the environment $V(x, y, z)$ during the time Δt. This interaction with the environment always causes surface damages to originate and accumulate in the system elements since *t*, *V*, *E* are variable. The system itself is also characteristic for the force interaction of its elements ($A \rightleftarrows B$). This means is that not only surface damage, but also volume (internal) damage should arise and develop, since the forces of such interaction are distributed over the volume of the elements and vary with time. Therefore the life (longevity) of the system is shown in (146) and (147) by the wavy lines. *Accumulation of irreversible surface and volume damages is the softening process which eventually causes the system degradation and fracture*.

Assume that the *system elements*, as well as the *entire system reveal the hardening property*, i.e., the ability to increase its resistance by both external and internal influences when they are hardened. Then the outcome of *struggle of opposites* (i.e., *hardening-softening processes*) also determines the *longevity* of the system, or its *lifetime* (operation). If the damage level increases in time, then the system degrades inevitably, as soon as the damage reaches some limiting (or critical) value.

Thus the writing

$$\forall \otimes \mathfrak{R} \otimes R \quad \overset{t_k,\ V_k}{\underset{E_k}{\downarrow}} \tag{148}$$

means the following. The degradation of the system is the process that leads to its disintegration within the volume

V_k in the time t_k into fragments (($\boldsymbol{V}$, $\boldsymbol{g}$)) and residues R. The degradation is accompanied by the release of the energy E_k; the fragments and residues dispersed in time and space constitute a set ($\otimes$) of disintegration products.

The products of the system degradation are represented by three components in expressions (145) and (148). First, these are the modified bodies A and B (denoted as $\boldsymbol{V}$ and $\boldsymbol{g}$, respectively). Secondly, these are system residues (denoted as R). In other words, $\boldsymbol{V}$ and $\boldsymbol{g}$) are the recognizable parts (fragments) of the disintegration products of the system since A and B are their images. As far as R is concerned, it is the unrecognizable (or hardly recognizable) part of the disintegration products of the system. This part can be represented as the one consisting of at least four components:

$$\boldsymbol{R = R\,(R_A^g,\, R_B^V,\, R_{A,V}^{t,V},\, R_{B,g}^{t,V})} \qquad (149)$$

i.e., $\boldsymbol{R_A^g}$ are the A residuals embedded in $\boldsymbol{g}$ and trapped by it. $\boldsymbol{R_B^V}$ are B residuals in $\boldsymbol{V}$, i.e., these are the B fragments embedded in $\boldsymbol{V}$ and trapped by it. $\boldsymbol{R_{A,V}^{t,V}}$ are the A and $\boldsymbol{V}$ residuals dissipated in the space (environment) V and in the time t. Finally, $\boldsymbol{R_{B,g}^{t,V}}$ are the B and $\boldsymbol{g}$ residuals dissipated in the space V and in the time t.

Residuals and fragments are going in the future ($+t$). Their existence is shown in (145) by the dashed arrow. This existence can be separate and is marked by the commas between the symbols $\boldsymbol{V}$, $\boldsymbol{g}$, R.

Expression (145) cannot be read or represented as some mathematical model. It should be understood as the conventional (symbolic) writing of the sequence of interrelated processes of system origination, existence, and degradation.

As the simplest specific example, consider one of the widespread active systems: *crankshaft journal* (A) – *sliding bearing* (B) of the rod head of the engine. Our interest is the longevity of the system.

The technological process of manufacturing parts A and B ends in the assembly $A \Leftrightarrow B$ – it is the process of system origination (146). Obviously, it is implemented in the time t_0 within the volume V_0 at the energy expenditure E_0. Then the life of the system (147) begins: run-in, normal operation, gradual loss of efficiency. In the course of the life the system $A \Leftrightarrow B$ changes into $A \leftrightarrows B$), i.e., assembly components undergo wearing at the contact pressure q and wear-fatigue damages accumulate in the crankshaft journal when acted upon by cyclic stresses σ. This occurs when the energy $E(V, t)$ interacts with the environment (oxidation of friction surfaces) during the entire lifetime Δt. Thus, both the environment $V(x, y, z)$ and the interaction energy E vary with time. The damage accumulation causes the system to degrade according to (148) and, hence, its failure (wear-fatigue fracture of the crankshaft journal, frictional fracture of bearing inserts). The system undergoes failing in the environment V_k in the time t_k followed by the release of the energy E_k. In the process of failure (148), the $\boldsymbol{V}$ and $\boldsymbol{g}$ fragments – the parts of the shaft A and the inserts B. Also R residuals – the wear products (149) are formed: the crankshaft particles embedded into the sliding bearing inserts ($\boldsymbol{R_A^g}$); the insert particles embedded into the crankshaft journal surface ($\boldsymbol{R_B^V}$); the products of surface damage of the crankshaft journal ($\boldsymbol{R_{A,V}^{t,V}}$) and the inserts ($\boldsymbol{R_{B,g}^{t,V}}$) scattered in the environment in the time t, i.e., the wear products removed from the friction zone.

As seen, based on (145) a sufficiently general and correct qualitative analysis of interactions of the system elements and the system with the environment is given.

The outlined qualitative picture can serve as a basis, for instance, for setting and describing quantitatively the longevity N (resource) of the active system. It is obvious that $\Delta t = N$ is the function of cyclic stresses σ in the crankshaft journal, the contact pressure q in the tribocoupling, the wear rate I of system elements, the accumulation rate of wear-fatigue damage ϑ, the properties (composition, structure) of the environment C_V and the elements A, B of the system (C_A, C_B):

$$N = N(\sigma, q, I_\sigma, \vartheta, C_V, C_A, C_B, \ldots).$$

This equation for longevity can be specifically implemented, for instance, using the methods of Applied Mechanics.

Similarly, the processes of origination, life, and degradation of other systems, for example, *solid – fluid*, etc. can be described. Differences will be only in specifying what interaction forces are implemented in the investigated case and what damages arise and develop.

If the *biological system*, for example, cardiovascular or musculoskeletal is considered, then a sufficient qualitative description of its life, damage, and degradation can be made with the use of the same symbolic model (145) developed as applied to inorganic active systems. Further, it is necessary to take into account a *specific complex of biological phenomena and factors* [61, 62]. It is shown that approach (145) can also be used to describe the general processes of birth, life and, death of a *living organism* that together with environment and habitat conditions forms the most complex *living system* in it. For this case the concept of Tribo-Fatigue life as a special method of damage accumulation [62] is developed.

Approach (145) is also used for description of the evolution of the MTD system, including in the translimiting state. Table 2 contains this approach with regard to the above-described diverse characteristics of system damage. It is obvious that the qualitative representation (145) of the evolution is supplemented here with the specific numerical analysis – at all nodal points of development (states A, B, C) and degradation (states C, D, E).

The general classification of the conceivable states of a system (object) in terms of damage is contained in columns 1, 2, 3. It is similar to the Table 1, but with the specification (as marked above) that the level of supercritical damage (ω_Σ^*) is assigned the superscript that means such a state. Table 2 also contains the appropriate physical characteristics of system states (column 5) and the additional analysis (column 4) based on the characteristic of its integrity ($\delta = 1 - \omega_\Sigma$).

Column 6 contains the symbolic description of all system states. The above-described energy states of the system are based on conditions (31) and (32) and contain two uncertainties. These uncertainties are interpreted as follows. When $\psi_D^* \to \infty$ (according to (31), the absolute average size (d_ψ^*) of particles forming during the system fracture must become arbitrarily small ($d_D^* \to 0$) by condition (32).

Table 2 – **Characteristics of the damage evolution of the MTD system**

MTD system states		Parameters		State properties (physical)	Symbol description of states	Energy conditions of states	Technogenic situations and possible damages
Symbol	Characteristic	damage	integrity ($\delta = 1-\psi$)				
1	2	3	4	5	6	7	8
A	Undamaged	$\omega_A = 0$	$\delta_A = 1$	Maintaining the integrity (size, shape, mass), structures (skeleton) and support (implementation) of all functions	V_0=const $A_0 \rightleftarrows B_0$ $u_\Sigma^{eff} = 0$	$u^{eff} = 0$ $\Psi_\Sigma = 0$	Failures (e.g., short-time reversible change of function)
B	Damaged	$0<\omega_B<1$	$1>\delta_B>0$	Development of complex damage and malfunctioning	$V_{ij} > 0$ $A \rightleftarrows B$ $u_\Sigma^{eff} > 0$	$u_\Sigma^{eff} < u_0$ $\Psi_\Sigma < 1$	Incidents (e.g., permissible system wear)
C	Critical (limiting)	$\omega_\Sigma=1=\omega_C$	$\delta_C= 0$	Total functional loss, multicriterion limiting state	$C \in (V \otimes g)$	$u_\Sigma^{eff} = u_0$ $\Psi_\Sigma = 1 = \Psi_C$ $d_c = 1$	Accidents (e.g., fatigue failure of engine shaft)
D	Supercritical	$1<\omega_D^*<\infty$	$\delta_D< 0$	Formation of multiple fragments, dissipated fragments and residuals	R_A^g, R_B^V $R_{A,V}^{t,V}$, $R_{B,g}^{t,V}$	$1> d_D^* > d_{\hbar\!\!\!/}$	Catastrophes (e.g., mid-air collision)
E	Disintegration (breakdown)	$\omega_E^* \to \infty$	$\delta_E \to -\infty$	Formation of nanoclusters, scattered atoms, elementary particles	✷	$d_E^* \to d_{\hbar\!\!\!/}$	Cataclysms (e.g., nuclear explosion)

Table 2 reveals these uncertainties (column 7). Namely, it is assumed that supercritical states are described by the changes in the size of particles forming within the range

$$1 > d_D^* > (1/k),$$

where the left restriction is determined by unity (as the symbol of the "single whole"), and the right one – arbitrarily (or infinitely) large integer k that lies within the limit

$$\lim_{k\to}(1/k) = \min d_D^* = d_{\hbar\!\!\!/} \approx 10^{-k}, \qquad (150)$$

where the conventional, yet finite quantity ($\hbar\!\!\!/$) is introduced as the limit of a possible growth of the integer k to the quantity ($k = \hbar\!\!\!/$) that can be specified as the total quantity of atoms in the system under investigation. In principle, it can be calculated if the size $d_{\hbar\!\!\!/}$ of atoms is known for materials, of which the system is "made"; thus $d_{\hbar\!\!\!/} \approx 10^{-k}$. In (150) it is then considered that the system fracture means its disintegration into such a "quantity" of particles that is equal to the initial number of atoms available in the system. The latter can be reasonably calculated practically for any systems. It has been established, for example, that the amount of atoms in the Universe approximately equals 10^{67} [38].

Thus, the growth of the level of *translimiting damage of a body* $\omega_\Sigma^*, \psi_u^* > 1$, (column 7, Table 2) signifies an appropriate decrease in the characteristic size of forming particles. Thus, the "location" of these particles is not specified – it can be any. But, naturally, it is meant that all particles will be finally spent for construction of those or other new systems (i.e., not necessarily – one system) [2]. This means that the reproduction of systems is inevitably implemented after their degradation – but, of course, in new conditions with new initial parameters.

Note: a significant drawback of the performed analysis is the absence of the determining parameter – the time t.

As applied to the specific MTD system, Figure 7 explains our idea of the time of its existence. The general concept of the *unidirectional time arrow* is borrowed from Thermodynamics (mostly from Physics). Thus, the question about the *nature of time* is not being discussed here (as in Physics and Philosophy). Further, according to Item XIV, it is assumed that the *existence time of the system under examination is always finite* and is defined by the interval $(0, T_*)$ where T_* is the time before the system disintegration (Table 1). Within this interval the time of its disintegration (failure) $T_\oplus << T_*$ is really defined. The failure of the system is interpreted as usual: it means the total loss of system functions and properties, which corresponds to the fact that the damage measure (for example, ω_Σ or ψ_Σ) reaches the limiting (critical) value $\omega_c = 1 = \psi$. At the moment of failure the system, therefore, ceases to exist as the whole. In Figure 7 it is shown that the existence of the system under study corresponds to a certain time interval on any its more general scale – for the Earth, the Solar system, the Universe (it is marked by the vertical arrows which separate the *past* and the *future*).

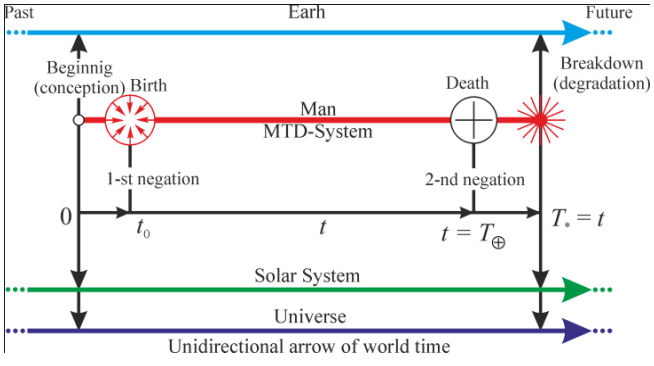


Figure 7 – Lifetime of the material system

Now describe the evolution of the MTD system.

Fig. 8 illustrates that based on the mechano-thermodynamic viewpoint, the A-evolution in time (Table 1) is implemented in *two stages*. The stage *ABC* ($\omega_\Sigma = \psi_u^{eff} \to 1$) is the *lifetime of the system as the integrity* when it performs all its functions. It is represented as the *development accompanied by an inevitable growth of damage and the deterioration of some functions* up to the moment when the limiting (critical) state is reached at the point *C*. *At this point the system completely "loses" all its functions*, for example, the accident (either the splitting of one of the system elements into 2 parts, or the unacceptable (limiting) wear in system, etc.). The *second stage CDE* then occurs and is *represented as the degradation process accompanied by the occurrence of numerous and various translimiting states* caused, for example, by catastrophes, explosions, cataclysms, etc. Example: at the point *C* the vessel under static pressure is divided into 2 parts; it disintegrates into a large number of "infinitely small" particles, if the nuclear explosion (point *E*) is implemented in it; it collapses into fragments if the explosion in it is initiated by a different quantity of explosive substances (line *CDE*). The moment of body disintegration into atoms (elementary particles, etc.) is denoted by the symbol (✺).

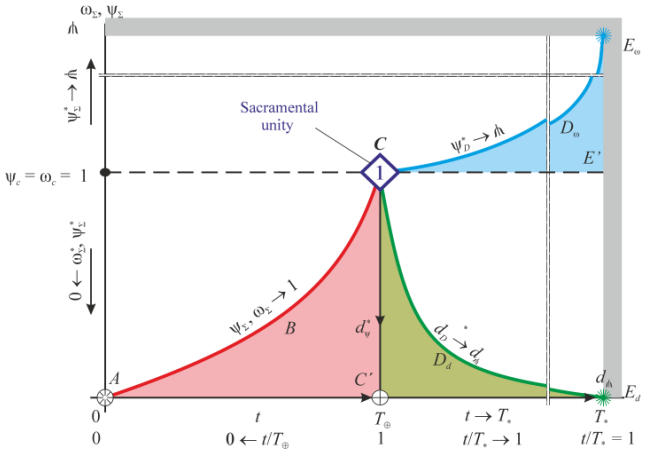


Figure 8 – Hypothesis of the MTD system evolution

The *second stage* (*translimiting states*) can be described in two ways. Either the analysis of the *average size* $d_D^* \to d_{\not\Lambda}$ of *disintegration particles* is used (formulas (142) and (143)) and represented by the curve CD_dE_d in Figure 8 (note that in this case the origin of coordinates is shifted to the point *C* and the size d_D^* ranges from 1 to 0 (line *CC′*), or the damage analysis $\psi_u^{eff} \to \not\Lambda$ is used and represented by the curve $CD_\omega E_\omega$ in Figure 8 (remind that here the "number of damages" corresponding to the disintegration (breakdown) of the system is designated by the number ψ conventionally equal to the number of atoms in the system.

The potentiality of the *parametric analysis* appears to be interesting and beneficial.

In our opinion, the representations as set forth above do not contradict the known and approved theories and the experimental results.

From Figure 8 it is possible to find two important *features of system A-evolution by time in terms of damage*. *First feature*: the plot reveals the *sacramental point C*, at which *three special – critical units* $\psi_u^{eff} = \omega_\Sigma = 1 = \omega_c$, $t/T_\oplus = 1$ and $d_\psi = 1 = d_c$ "come together". It is the *evolution epicenter*, or its *apotheosis*. These critical units also define the *"division"* of A-evolution into two essentially differing stages – *development stage ABC* and *degradation stage CDE*. It is the point when the system loses all its functions, i.e., the point of transition to various translimiting states.

Second feature: using the plot in Figure 8, obviously, it is possible to describe and define the *effective energy conservation law*

$$\int_0^{T_\oplus} u_\Sigma^{eff}(t)dt \equiv \int_{T_\oplus}^{T_*} u_{eff}^{\Sigma}(t)dt \qquad (151)$$

where u_Σ^{eff}, u_{eff}^{Σ} is the effective energy on the first (development) and on the second (degradation) stages, respectively. The statement of this law is as follows: *effective energy absorbed by the system in the process of reaching the limiting (critical) state is identically equal to the released (scattered) effective energy in the process of its degradation up to disintegration* (for example, into atoms).

Geometrically, this law requires the equality of the three areas in Figure 8

$$S_{ABCC} \equiv \phi S_{C'CD_dE_d} \equiv S_{CD_\omega E_\omega E'}$$

where ϕ is the function of parameter transformation [for example, according to (143)].

From the above-stated three main conclusions follow:

1 *Damages are the fundamental physical property (and the functional duty) of any system and all of its elements*.

2 *Damage of each object (any existing one) inevitably grows up to its breakdown – decomposition (disintegration) into a set of particles of arbitrarily small size, i.e., it is the unidirectional process of time*:

$$\begin{cases} \Psi_\Sigma^U = \Psi_\Sigma^U\left(\sigma_n^{(V,W)}, \varepsilon_n^{(V,W)}, T_\Sigma, V_{ij}, V_{ijT}, V_0, Ch, \Lambda_{i\backslash j}, m_k, t, u_0\right) \Rightarrow \not\Lambda, & (152) \\ d_D^* \overset{\bar{t}}{\Rightarrow} d. & (153) \end{cases}$$

3 *Not only the unity and struggle of opposites but also the directivity of various and complex physical processes of hardening-softening (depending on the level of loads and time) are typical of the evolution of the system by damage. It means that the Λ-function of damage interactions (of all kinds) can take three classes of values*: *a*) $\Lambda < 1$ when the hardening process is dominant; *b*) $\Lambda > 1$ when the softening process is dominant; *c*) $\Lambda = 1$ when a stable hardening to softening process ratio is found.

Thus, the *first law of Mechanothermodynamics* states that for evolution of any system the unidirectional process of its damage and disintegration, finally, into an infinitely large number of small components (fragments, atoms, elementary particles, etc.) is inevitable. In fact, it is equivalent to the recognition of the thesis on the unending of evolution if it is taken into account that disintegration products of any system become a construction material for new systems. Thus, the evolution hysteresis is formed. Generalizing, it may be said that our Universe is indestructible as it evolves by damage. This corresponds to the philosophical concept that matter and motion are eternal, and damage is the fundamental property (and a duty) of all systems, including living and intelligent ones [1, 31, 61].

The *second law of Mechanothermodynamics* states: Λ-functions of interactions must take three classes of values ($\Lambda \gtrless 1$) to describe not only the unity and struggle, but also the directivity of physical hardening-softening processes in the system, i.e., the system evolution by damage [1, 31].

Generalization of the results presented above with regard to [1, 2] is shown in Figure 9. It can be seen that the state of a system can be equivalently described in terms of either energy or entropy.

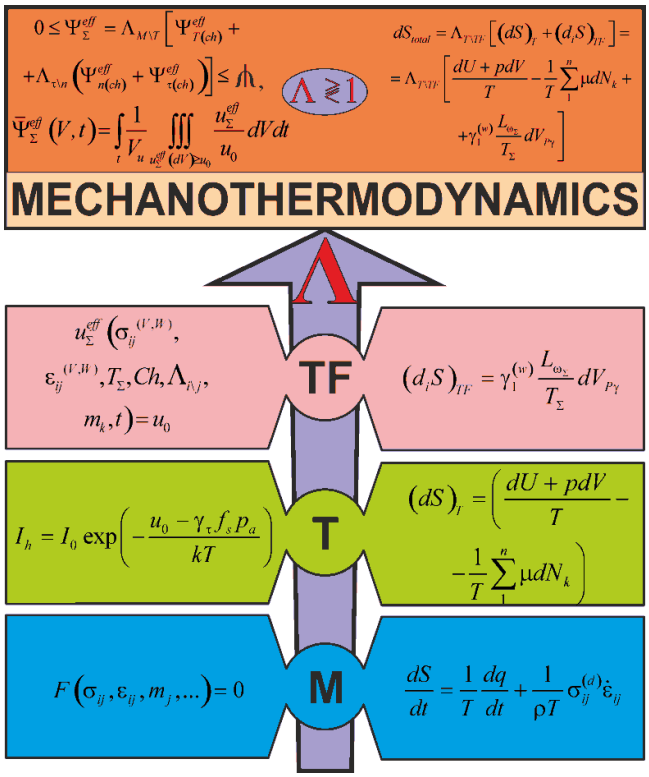


Figure 9 – Energy (left) and entropy (right) approaches to the development of Mechanothermodynamics (M – Mechanics, T – Thermodynamics, TF – Tribo-Fatigue)

The main drawback of such descriptions is the known unreality of energy and, hence, of entropy: physical energy carriers are not detected and, apparently, do not exist. As R. Feynman [64] said figuratively, they cannot be touched. Damages are quite a different matter: they are physically real, can be touched, actually define any of the conceivable states of material bodies and systems. Kinetic process of their accumulation, as well as the time stream is inevitable and unidirectional. If Mechanothermodynamics considers the damage of a system as its fundamental physical property (and duty), one can hope that based on it, the consistent general theory of evolution of any systems, including living and intelligent, can be created. For instance, the idea of life as of a special method of damage accumulation (biological, mechanical, intellectual, etc.) is being developed in [32, 61].

Thus, the attempt was made above to formulate the basic provisions of a new (or, better to say, integrated) physical discipline – Mechanothermodynamics with the use of the energy principles. This discipline combines two branches of Physics in effort not to argue or not to compete with each other, but to take a fresh look at the MTD system evolution (Figure 10).

From Figure 11 it follows that the principles of Mechanothermodynamics can be formulated in two ways: 1) Mechanics → Tribo-Fatigue → Mechanothermodynamics and 2) Thermodynamics → Tribo-Fatigue → Mechanothermodynamics. Thus, Tribo-Fatigue has become a bridge for transition from Mechanics and Thermodynamics to Mechanothermodynamics. The fact that the both ways are leading to one purpose and, finally, yield the same (unified) result, means that the above-mentioned two methodologies of analysis are valid, correct and do not contradict each other.

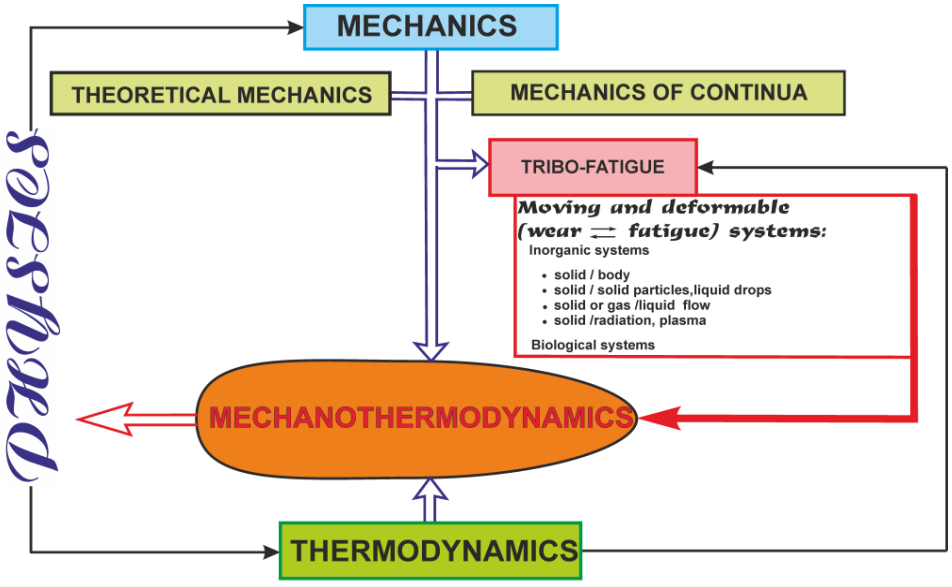


Figure 10 – Ways toward Mechanothermodynamics as the new branch of Physics

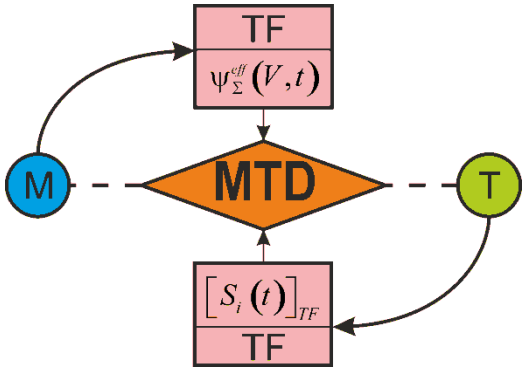


Figure 11 – Tribo-Fatigue bridges from Mechanics (M) and Thermodynamics (T) to MTD Mechanothermodynamics (the solid lines with arrows; the dashed lines show the unrealized ways (for more than 150 years) from M or T to MTD)

Returning to Figure 1, it is seen that it is ended with the arrow with a question: what sort of object will be behind the MTD system? The obvious and common answer is: it is our real world. Nowadays it is being studied actively by numerous and various sciences – from Chemistry and Biology... through Mechanics and Thermodynamics... and up to Philosophy; from all points of view. From Figure 1 it is clear that the MTD system should be followed by an object that is somewhat more complex (however simpler) than the real system, for example, the MTD system with some "elements of intelligence". The first works in this area of research are already available [65–67].

Analysis and generalization of experimental data. Experimental verification of generalized criterion (72) of the limiting state of a MTD system is extremely difficult because of the absence of relevant experimental data. Their acquisition is though very relevant but at the same time is very difficult and expensive. Therefore, in this paper, the analysis of the particular solution of criterion (77) in form of (78) is given.

From (78) it is easy to obtain a number of formulas important for application. So, the conditions of purely thermal (or thermodynamic) damage (when $\sigma = 0$ and $\tau_w = 0$) or purely mechanical damage (when $T_\Sigma \to 0$) will be as follows:

$$a_T T_\Sigma = u_0; \qquad (154)$$

$$\Lambda_{n\backslash\tau}\left(a_n\sigma^2 + a_\tau\tau_w^2\right) = u_0. \qquad (155)$$

For isothermal mechanical fatigue (when $\tau_w = 0$) we have

$$\Lambda_{M\backslash T}\left(a_T T_\Sigma + a_n\sigma^2\right) = u_0, \qquad (156)$$

and for isothermal frictional fatigue (when $\sigma = 0$) we obtain

$$\Lambda_{M\backslash T}\left(a_T T_\Sigma + a_\tau\tau_w^2\right) = u_0. \qquad (157)$$

The general analysis of the above-described partial criteria allows three main conclusions to be made.

1 The growth of loading parameters (σ, τ_w, T_Σ, D) results in the corresponding acceleration of reaching the limiting state (u_0).

2 The limiting state of the system can also be reached by increasing only one (any) of the loading parameters (when maintaining the same values of other parameters).

3 If $\Lambda > 1$, the damageability of the system accordingly enhances (i.e., the processes of its softening are dominant), and if $\Lambda < 1$ it slows down (i.e., the processes of its hardening appear to be preferable) in comparison with the damage due to the joint action of loading parameters alone (with no regard to the dialectic interaction of irreversible damages).

The last conclusion is also the result of a fundamentally new approach to the construction of the criterion of the limiting state of MTD systems [67]. According to this approach, *not the mutual influence of the factors, but the interaction* ($\Lambda \gtrless 1$) *of phenomena determines the damage processes in the MTD system* [1, 31–33, 67]. In this regard, the paper synthesized the results of more than 600 diverse experimental data. This permitted the generalized MTD function of critical by damage states to be revealed.

Refer to one of the special cases of criterion (78) – *isothermal mechanical fatigue*. From (156) it follows that

$$\log\sigma_{-1T} = \frac{1}{2}\log C_T; \quad C_T = \left[u_0/\Lambda_{M\backslash T} - a_T T_\Sigma\right]\cdot\frac{1}{a_n}. \qquad (158)$$

According to (158), the dependence of limiting stresses on the parameter of *thermomechanic resistance* C_T in the double logarithmic coordinates is to be a straight line with the angular coefficient (1/2). The general regularity is as follows: *the higher the value of the parameter* C_T, *the greater is the quantity* σ_{-1T}. Figure 12 shows a satisfactory evidence of this dependence for numerous different-grade steels tested for fatigue in different conditions [57, 59, 68]. It is seen that the C_T value varied by more than two orders, i.e. by a factor of 100 or more, and the values of the endurance limit σ_{-1T} – by more than two orders, i.e., by a factor of 10 or more, thus the testing temperature varied in the range from the helium temperature to $0.8T_s$ (T_s is the temperature of melting) . As shown in Figure 12, equation (158) adequately describes the results of more than 150 experiments.

Equation (158) is also checked for different-class metal materials according to the fatigue results that have been obtained by many authors and are illustrated in Figure 13, *a*. In [57, 59] it is possible to find the list of literature references. In Figure 13, *b*, the similar analysis is made on the basis of the test results for tension at different temperatures (σ_{uT} is the stress limit). In this case, it is taken that $\sigma_{-1} = \sigma_{uT}$ in equation (158). It is obvious: the correlation coefficient is very high – not less than $r = 0.722$ (very occasionally), but in the most cases it exceeds $r = 0.9$; the analysis includes more than 300 test results. Works [57, 59] contain other examples of successful experimental approbation of criterion (158). This allows us to hope that even more general criteria (for example, equations (77) and (78)) will appear to be practically acceptable. In our opinion, further studies must confirm our hope.

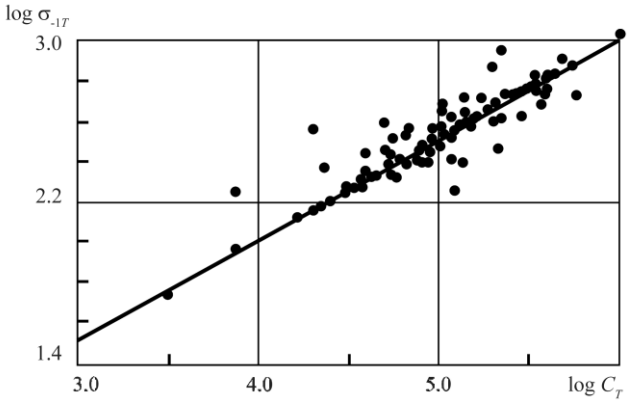


Figure 12 – Endurance limits of constructional steels vs. the parameter C_T (*A. V. Bogdanovich., L.A. Sosnovskiy*)

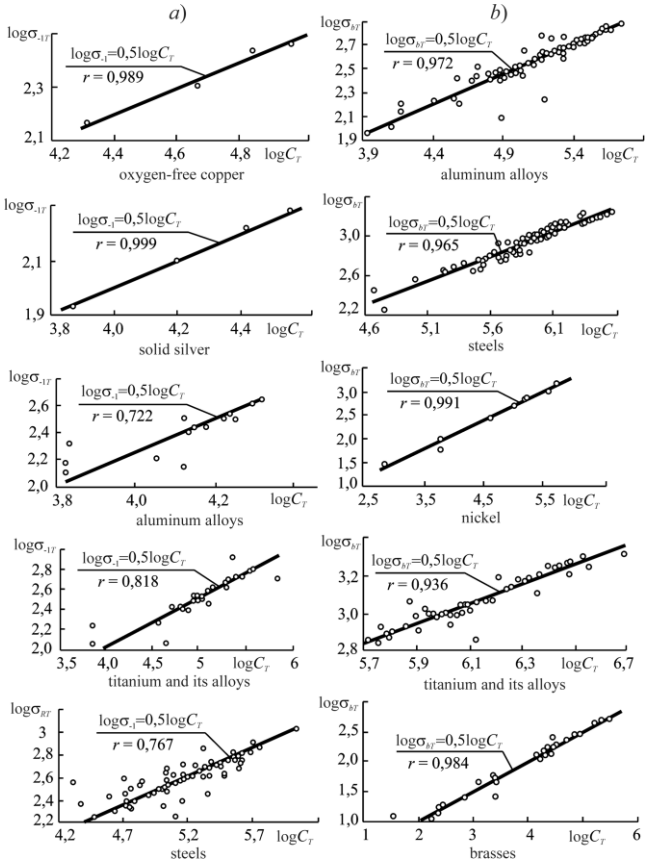


Figure 13 – Dependences $\sigma_{-1}(C_T)$ (*a*) and $\sigma_u(C_T)$ (*b*) for various metal materials (*A. V. Bogdanovich., L. A. Sosnovskiy*)

As defined above, criterion equation (155) is valid for $\sigma \le \sigma_u$, and depending on the testing conditions, τ_W can be interpreted as the largest contact pressure (p_0) at the center of the contact area at rolling. It can also be interpreted as the sliding stress (τ_w) or as the average (nominal) sliding pressure p_a in the contact zone, or as the fretting contact pressure (q). If the value $\sigma = \sigma_{-1}$ is fixed and $\sigma_{-1} \ll \sigma_u$ then equation (28) can be represented in the form of the diagram of the limiting states of Tribo-Fatigue systems [2, 57, 68] (Figure 14), in which the zones of realization of spontaneous processes of hardening-softening ($\Lambda \gtrless 1$) are clearly distinguishable. So, Figure 14 yields the above-mentioned obvious conclusions: if $\Lambda < 1$, then we are dealing with the self-hardening system (during tests or during the operation under these conditions); if $\Lambda > 1$, then the system turns to be self-softening; if it is found that $\Lambda < 1$ converts to $\Lambda > 1$, then it implies that owing to the changes in the determining conditions of operation or use, the hardening processes are replaced by the softening processes.

Additional experimental support for these conclusions is provided in Figures 15–17. Note that for spontaneous hardening (for $\Lambda < 1$, Figures 14–16) it appears that the *stress limit in wear-fatigue tests is higher, than in routine fatigue tests. It means that in these conditions the processes of friction and wear become "useful"*. There are numerous works (for example, [69]), according to which dosed wear in real Tribo-Fatigue systems (for example, wheel/rail) leads to the appropriate growth of their fatigue strength. Vice versa, at $\Lambda >> 1$ (Figure 17) they lead to a strong damage growth: the fatigue limit decreases with increasing the contact pressure q by factor of 2–3. In addition, there are many works (for example, [70]), according to which the system wear results in a sudden decrease of fatigue strength.

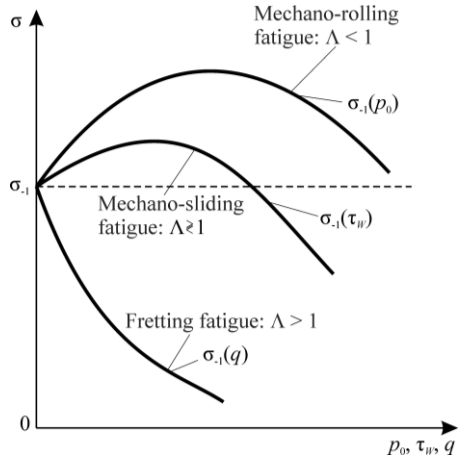


Figure 14 – Diagram explaining the basic features of Λ-interactions in the Tribo-Fatigue system

Tables 3 and 4 summarize the physical signs of different (often encountered in practice) signs of the limiting state that can find use in the corresponding research areas.

As for the determination of the parameters $\Lambda_{M\backslash T}$ and $\Lambda_{n\backslash\tau}$ it is shown in [2, 57, 59] that, for example, the parameter $\Lambda_{n\backslash\tau}$ is the *function of the relative skewness coefficient of wear-fatigue damage*:

$$\overline{\rho}_{n\backslash\tau} = \left(\frac{\tau_w}{\tau_f}\right)^2 \left(\frac{\sigma_{-1}}{\sigma}\right)^2 \qquad (159)$$

Hence, it follows that $\overline{\rho}_{n\backslash\tau}$ depends not only on the absolute values of effective (σ, τ_w) and limiting (σ_{-1}, τ_f) stresses, but also on their ratios, namely: τ_w/σ, σ_{-1}/τ_f, σ_{-1}/σ, $\tau_w/\tau_f \gtrless 1$. This means, for example, that significantly different regularities of irreversible damage accumulation will be implemented depending on the realization of this or that of the inequalities $\sigma \gtrless \sigma_{-1}$, $\tau_w \gtrless \tau_f$. This conclusion corresponds to the known experimental results and theoretical models. Figure 18 shows the analysis with regard to the possible dependences $\log\Lambda_{n\backslash\tau} - \log\overline{\rho}_{n\backslash\tau}$ [2, 59]. A more detailed analysis of the interdependences $\Lambda_{n\backslash\tau}(\overline{\rho}_{n\backslash\tau})$ can be found in [2, 57, 59].

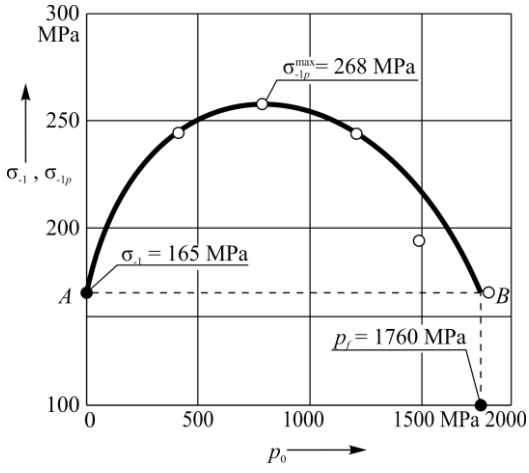


Figure 15 – Influence of rolling friction on the resistance of contact-mechanical fatigue during the tests of the Tribo-Fatigue steel 45 (shaft)/steel 25 KhGT (roller) system (*L. A. Sosnovskiy, S. A. Tyurin*)

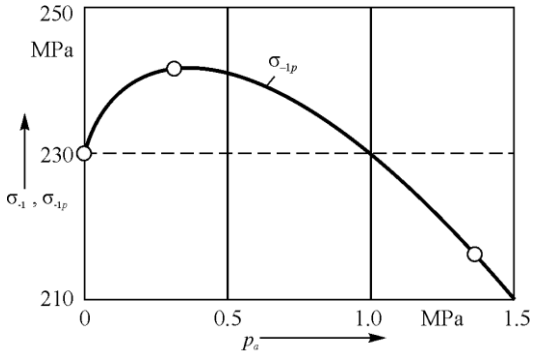


Figure 16 – Limiting stresses vs. the contact pressure for the tribo-fatigue steel 45 (shaft)/cast iron (partial bearing insert) system (*V. I. Pokhmursky et al.*)

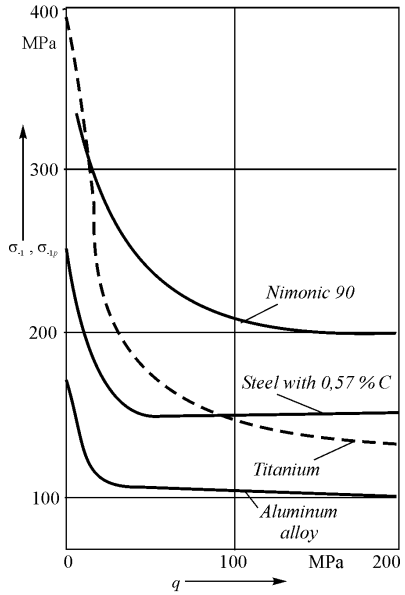


Figure 17 – Contact pressure vs. changes in the fatigue limit at fretting fatigue according to R. B. Waterhouse (nimonic-90 – *Harris W. J.*; steel with 0.5% C – *Peterson R. E.*; titanium – *Sinclair G. M. Liu H. W. Corten H. T.*; aluminum alloy – *Corten H. T.*)

Table 3 – **Set of main indicators of physical state**

Energy state		Condition of reaching the limiting (critical) state
Symbol	Physical state and its characteristic	
M	Mechanical state σ_{ij}	$u_n^{eff} \xrightarrow{\sigma_{ij}\to\sigma_{lim}} u_0$
T	Thermodynamic state T_Σ	$u_T^{eff} \xrightarrow{T_\Sigma\to T_S} u_0$
MTD	Mechanothermodynamical state σ_{ijT} , T_Σ	$u_\Sigma^{eff} \Rightarrow u_0$ ($\sigma_{ijT}\to\sigma_{lim}(T)$; $T_\Sigma\to T_S$)
tMTD	Mechanothermodynamical state in time σ_{ijT}, T_Σ, t	$u_\Sigma^{eff} \Rightarrow u_0$ ($\sigma_{ijT}\to\sigma_{lim}(T)$; $T_\Sigma\to T_S$; $t\to t_{lim}$)
Nomenclature: σ_{lim} is the limiting stress; T_s is the melting temperature; t_{lim} is the longevity; σ_{ij} is the stress (strain) tensors; T_Σ is the temperature due to all heat sources; σ_{ijT} is the stress tensor in the isothermal (T_Σ = const) state; σ_{ijT}, T_Σ is the stress-strain state and the thermodynamic state, respectively; σ_{ijT}, T_Σ, t is the stress-strain state and the thermodynamic state in time, respectively		

The plot of the $\Lambda_{T\backslash M}$ interactions on the parameter $\bar{\rho}_{T\backslash M}$ can be analyzed in a similar way. Such a plot of steel, aluminum alloys, and nickel (according to the extensive experimental results [2, 57, 59]) in the double logarithmic coordinates is shown in Figure 19. The correlation coefficient has appeared to be very high: from $r = 0.862$ to $r = 0.999$. The plot of $\Lambda_{T\backslash M}(\bar{\rho}_{T\backslash M})$ as a rule, undergoes sudden changes for $\log \bar{\rho}_{T\backslash M} = 0$, i.e., at the value $\bar{\rho}_{T\backslash M} = 1$ when thermal and force damages appear to be equilibrium (as compared to the similar changes in the plots in Figure 18).

Table 4 – **Specification of characteristics and their physical signs of the limiting state**

Criterion	Condition of reaching the limiting state	Physical sign
*L*1	$\sigma_{lim} = \sigma_u$ σ_u – stress limit at tension	Static fracture
*L*2	$\sigma_{lim} = \sigma_{-1}$ σ_{-1} – mechanical fatigue limit	Fatigue fracture (into parts)
*L*3	$\sigma_{lim} = p_f$ p_f – limiting contact pressure at rolling	Pittings of critical density (critical depth), excessive wear
*L*4	$\sigma_{lim} = \tau_f$ τ_f – limiting stress at sliding	Limiting wear
*L*5	$\sigma_{lim} = \begin{cases}\sigma_{-1p}\\ \sigma_{-1\tau}\end{cases}$ σ_{-1p}, $\sigma_{-1\tau}$ – limiting stress during the direct effect implementation [2]	Fatigue fracture (into parts) depending on the contact pressure (index *p*) at rolling or on the friction stress (index τ) at sliding (direct effect in Tribo-Fatigue)
*L*6	$\sigma_{lim} = \begin{cases}p_{f\sigma}\\ \tau_{f\sigma}\end{cases}$ $p_{f\sigma}$, $\tau_{f\sigma}$ – limiting stresses during the back effect implementation [2]	Pittings of critical density (critical depth) or excessive wear at rolling or sliding depending on the level of cyclic stresses σ (index σ) (back effect in Tribo-Fatique)
*L*7	$\sigma_{lim} = \sigma_{-1q}$ σ_{-1q} – fretting fatigue limit	Fatigue fracture at fretting corrosion and (or) fretting wear
*L*8	$\sigma_{limT} = \sigma_{-1T}$ σ_{-1T} – isothermal fatigue limit	Limiting state depending on temperature (isothermal fatigue)
*L*9	$T_{lim} = T_s$ T_s – melting point	Thermal (thermodynamic) damage
*L*10	$t_{lim} = t_c$ t_c – longevity	Time (physical) prior to the onset of the limiting state on the basis of any sign

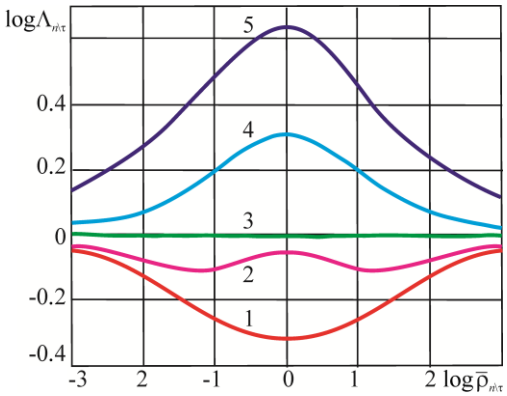


Figure 18 – Typical plots of the character and direction of hardening-softening processes ($\Lambda \gtrless 1$) vs. the skewness coefficient of the damage $\bar{\rho}$: 1, 2 – mechano-rolling fatigue; 2, 3, 4 – mechano-sliding fatigue; 4, 5 – fretting-fatigue

For steels and nickel at $\bar{\rho}_{T\backslash M} < 1$ the direct dependence is found between $\Lambda_{T\backslash M}$ and $\bar{\rho}_{T\backslash M}$, and at $\bar{\rho}_{T\backslash M} > 1$ it becomes

inverse. For aluminum alloys the dependence $\Lambda_{T\backslash M}(\overline{\rho}_{T\backslash M})$ is also direct, but it is located (at $\overline{\rho}_{T\backslash M} < 1$) in III quadrant.

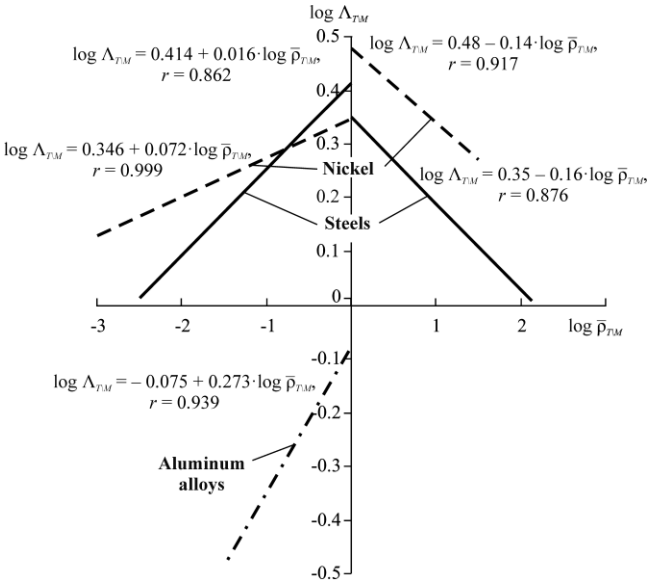


Figure 19 – Logarithmic plots of $\Lambda_{T\backslash M}(\overline{\rho}_{T\backslash M})$ built using the experimental data (*L. A. Sosnovskiy, A.V. Bogdanovich*)

Thus, it is experimentally confirmed that the *interaction parameter* $\Lambda_{T\backslash M}$ *is sensitive not only to the effective thermal-to-mechanical energy ratio, but also to the structure and composition (or nature) of metal materials*. The last conclusion is also valid for the parameter $\Lambda_{n\backslash\tau}$: its numerical values appear to be significantly different, for example, for metal/metal and metal/polymer active systems – even in the case when the ratios $\sigma\backslash\sigma_{-1}$ and $\tau_w\backslash\tau_f$ are identical for them.

The data of more than 600 tests of metals and their alloys (under isothermal conditions) obtained by many authors are briefly analyzed and have been presented above.

It was found that the thermodynamic dependence of limiting stresses can be represented in the $\log\sigma_{\lim} - \log C_T$ coordinates (Figures 12 and 13 and formula (158), where function

$$C_T = C_T\left(T,\, u_0,\, a_n,\, a_T,\, \Lambda_{M\backslash T}\right) \qquad (160)$$

is satisfactory under both the conditions of static tension ($\sigma_{\lim} = \sigma_u$) and fatigue fracture ($\sigma_{\lim} = \sigma_{-1}$) for numerous and different metal materials (steels; aluminum, titanium, and other alloys, etc.). In addition, interrelation (158) appears to be valid practically within the entire possible interval of temperature ($T \le 0{,}8T_S$) and stress ($\sigma \le \sigma_u$) varying with the correlation coefficient r = 0,7 in the specific cases and usually with r > 0,9. Then model (158) turns to be *fundamental* (Figure 20). This simplified model at first might seem to be questionable, since in the known literature [71 etc.] the explicit temperature dependence of limiting stresses is described by means of complex curves. This is attributed to the changes in the failure mechanisms of different materials under various testing conditions – at normal, operating, and other temperatures.

Nevertheless the fundamental nature of model (158) is supported experimentally (Figures 12 and 13).

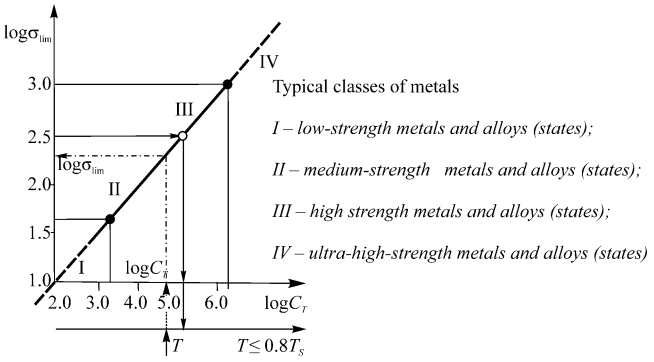


Figure 20 – Generalized (82) MTD function of limiting states of metals and alloys ($\sigma_{\lim} \le \sigma_u;\ T \le 0.8T_S$)

From the theoretical standpoint, the following considerations speak in favor of model (158). It has four parameters [formula (160)], and one of them (u_0) is a fundamental constant of substance (formulas (48), (49) in [67]), and two others (a_T, a_n) are defined by the boundary conditions as the ratios of u_0 and physical constants σ_d and T_d of this material [59]:

$$a_\sigma = u_0 / \sigma_d^2,\ a_T = u_0 / \sigma_d. \qquad (161)$$

The methods of σ_d and T_d determination are described in [2, 57, 59]. Here we remind that the material failure limit σ_d is determined under the tension conditions as $T_\Sigma \to 0$ and the failure temperature T_d – at the body heating at $\sigma = 0$. As can be seen from the above, the *dual character of accumulation processes of damage and failure* caused by 1) mechanical stress and 2) thermal activation of this stress in time [48] is considered in the general case. Finally, as briefly described above and outlined in [4, 48], the function $\Lambda_{M\backslash T} \gtrless 1$ considers the interaction of damages due to the changes in the ratios $\sigma \gtrless \sigma_{lim}$. In the known studies [2, 4, 72] it is also convincingly proved many times that just this ratio is responsible for the character and damage mechanisms at elastic, inelastic, elastoplastic and plastic strain. Also the role of thermal fluctuations ($T_\Sigma < T_d$) is, for example, studied in detail in [48, 49].

What is left is to put the "last point" on the argumentation in favor of the fundamental character of model (158). If it is really fundamental, then it must also be valid for nonmetal, for example, polymer materials – according to hypothesis (48) in [67]. The analysis results of the polymer tests based on the experimental data [73] are presented in Table 5 and Figure 21. It is obvious that model (158) is verified by the correlation coefficient r = 0.917. Note that the test results for not only "normal" samples (with a diameter of ~ 5 mm), but also for thin polymer threads and films are processed, not only at tension, but also at torsion and bending. A large deviation of several points from the fundamental straight line is due to the fact that for these results $\Lambda_{M\backslash T}$ = 1 is conventionally assumed thanks to the lack of experimental data to assess the real value of this parameter.

The generalized experimentally substantiated MTD function of the limiting states (in terms of damage) is shown in Figure 22. Relatively large deviations of particular experimental points from the predicted ones are also seen in Figures 12 and 13 for two reasons: either the available references have no data for a correct assessment of required parameters, or the conducted experiments contain significant errors or they are not quite correct methodically.

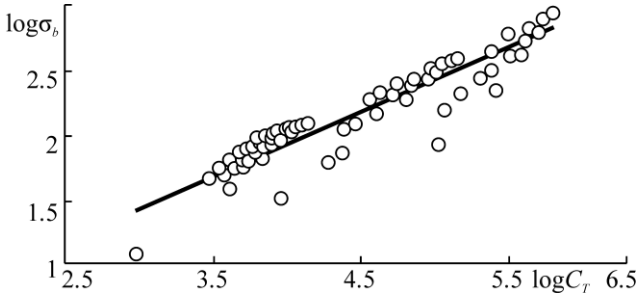


Figure 21 – Dependence $\sigma_u(C_T)$ for polymer materials (*A.V. Bogdanovich*)

Table 5 – **Analysis of the main characteristic of polymer materials on the basis of the experimental data [73]**

Material and reference	u_0, $\frac{\text{kJ}}{\text{mol}}$	$\frac{a_T}{a_n}$, $\frac{\text{MPa}^2}{\text{K}}$ $\left(\frac{\text{kJ}}{\text{mol}\cdot\text{K}} / \frac{\text{kJ}}{\text{mol}\cdot\text{MPa}^2}\right)$	Test data	
			$\frac{\text{K}}{\sigma_b,\ \text{MPa}}$	Sample size
Polyethylene high-density film (HDPF), grade 20806-024	108	$\frac{0.275}{2.94\cdot10^{-4}}$	$\frac{275...383}{32...386}$	5
Polypropylene film (PF) grade 03П10/005	119	$\frac{0.234}{1.70\cdot10^{-4}}$	$\frac{273...423}{150...570}$	5
Hardened staple fiber made of polyvinyl alcohol (PVA) "Vinol MF"	111	$\frac{0.227}{7.62\cdot10^{-5}}$	$\frac{273...453}{80...802}$	5
Thread based on perchlorvinyl resin (PCV) grade "Chlorine"	114	$\frac{0.285}{2.56\cdot10^{-4}}$	$\frac{273...383}{60...376}$	5
Caprone thread (PCA) (GOST 7054067)	169	$\frac{0.282}{1.68\cdot10^{-4}}$	$\frac{275...453}{300...740}$	5
Polyethylene terephthalate film (PET) (TU 6-05-1597-72)	222	$\frac{0.342}{9.82\cdot10^{-4}}$	$\frac{279...498}{200...362}$	4
Polyamide film PM-1 (TU 6-05-1597-72)	202	$\frac{0.297}{2.1\cdot10^{-3}}$	$\frac{273...673}{12...240}$	7
Polystyrol (PS) at bending	281	$\frac{0.627}{2\cdot10^{-2}}$	$\frac{77...290}{56...108}$	10
Polymetalmethacrylate (PMMA) at bending	277	$\frac{0.558}{1.74\cdot10^{-2}}$	$\frac{77...290}{66...116}$	10
High-impact polystyrene (HIPS) at tension and torsion	277 252	$\frac{0.699}{2.53\cdot10^{-2}}$ $\frac{0.636}{1.84\cdot10^{-2}}$	$\frac{77...290}{48...94}$ $\frac{77...290}{50...105}$	10 10

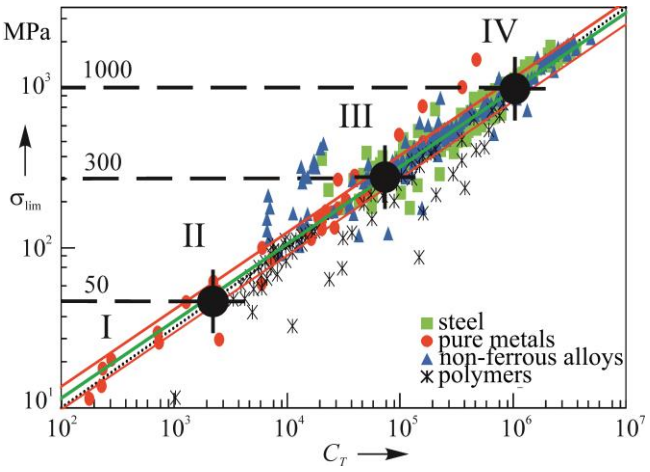


Figure 22 – Experimentally justified MTD function of critical by damage states of metal and polymer materials

Note that model (158) may seem to be non-fundamental because of its simplicity. However, remind the classic dictum: the fundamental dependence cannot be complicated (or: any law is described by the simplest formula.

Thus, model (158) can serve for prediction (shown by the arrows from T to $\sigma_{\lim}$ in Figure 20) of the *mechanical behavior of materials in the thermodynamic medium*:

$$T \xrightarrow[\overline{a_n, a_T}]{\underline{u_0,\ \Lambda_{M\setminus T}}} \log C_T \rightarrow \rightarrow \log\sigma_{\lim}\left(T, u_0, a_n, a_T, \Lambda_{M\setminus T}\right) \rightarrow \sigma_{\lim(T)}. \tag{162}$$

The state of medium in (162) is described by the parameters T, a_T, $\Lambda_{M\setminus T}$.

As it is seen the predictions by (158) and (162) are applicable for the materials of different nature and structure irrespectively of damage and fracture mechanisms at static and cyclic loading. It would be interesting to make a similar analysis of the tests at impact, but such an analysis lies outside the scope of the present work.

Certainly, due to the linearity of function (158), the *reverse prediction* appears to be possible and effective. If it is necessary to have a given mechanical state of the material (determined by the parameters u_0, $\sigma_{\lim(T)}$), then the requirements can be formulated to the medium (determined by the parameters T, a_T, a_n, $\Lambda_{M\setminus T}$) where the system can operate (in Figure 20 it is shown by the arrows from $\sigma_{\lim}$ to T):

$$\sigma_{\lim(T)} \rightarrow \log\sigma_{\lim(T)} \xrightarrow[\overline{a_n, a_T}]{\underline{u_0,\ \Lambda_{M\setminus T}}} \log C_T \rightarrow \rightarrow C_T\left(T, u_0, a_n, a_\tau, \Lambda_{M\setminus T}\right) \rightarrow T. \tag{163}$$

Note that the attempts to construct the explicit temperature dependence of limiting stresses in uniform, semilogarithmic and logarithmic coordinates for different materials and different testing conditions are quite ineffective (Figure 23).

We will further briefly analyze a *more complex problem of the MTD system operation in the medium* in which the processes of *thermal corrosion and corrosion at stress* are implemented. From (77) at $\tau_w = 0$ we have

$$\Lambda_{M\setminus T}\left(\frac{a_T}{1-D_T}T_\Sigma + \frac{a_n}{1-D_n}\sigma^2\right) = u_0. \tag{164}$$

Upon simple manipulations we obtain

$$\sigma_{\lim(T,ch)} = \frac{1}{2}\log C_{T(ch)}, \tag{165}$$

where, as can be easily shown, thermal resistance to corrosion at stress is:

$$C_{T(ch)} = C_{T(ch)}(T,\ u_0,\ a_n,\ a_T,\ \Lambda_{M\setminus T},\ v_{ch},\ v_{ch(\sigma)},\ m_{v(\sigma)},\ v_{ch(T)},\ m_{v(T)}). \tag{166}$$

It can be seen that laws (158) and (165) are fundamentally (and formally) identical and differ in the fact that appropriate functions (160) and (166) take account of those parameters which describe the damage processes characteristic for the phenomena analyzed. So, in (166) the parame-

ters v_{ch}, $v_{ch(\sigma)}$, $m_{v(\sigma)}$, $v_{ch(T)}$, $m_{v(T)}$ describe the processes of thermal corrosion at stress (formula (76) in [67]). Based on (165) and (166) it is easy to build prediction algorithms [like (162) and (163)] of resistance to corrosion at stress.

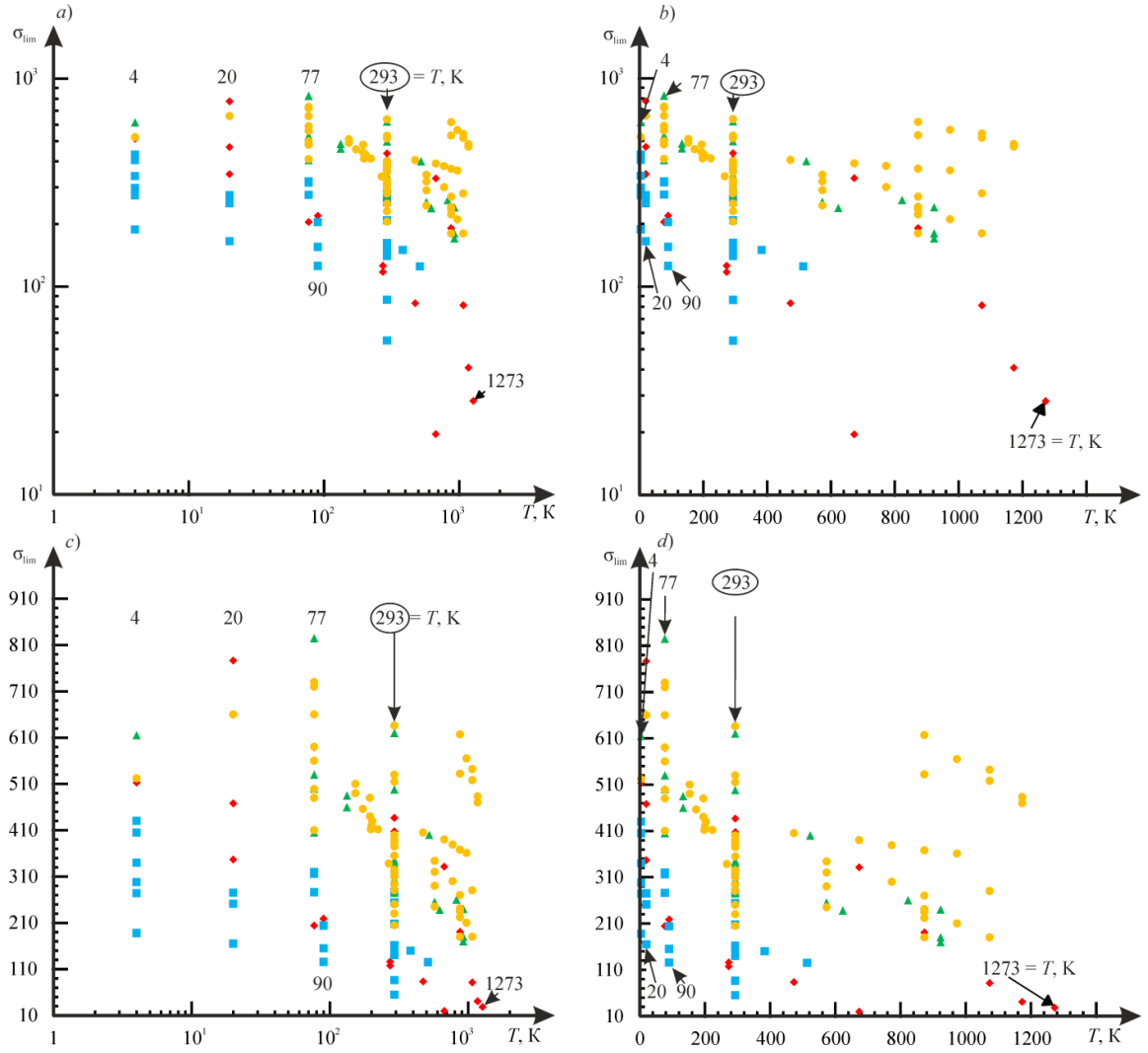


Figure 23 – Explicit temperature dependences of the fatigue limit for metal materials (based on 136 test results of various authors)

A further and detailed analysis of (165) and (166) is beyond the scope of this study.

Note that solutions (77)–(157) can be analyzed similarly for other testing (or operating) conditions.

Conclusion.

1 It is shown, that the creation of Mechanothermodynamics – a generalized physical discipline is possible by constructing two bridges. The first is Tribo-Fatigue entropy which paved the way from Thermodynamics to Mechanics. The second is a fundamental understanding in Tribo-Fatigue of irreversible damage of everything that exists which paved the way from Mechanics to Thermodynamics (Figures 9–11). The first and the second principles of Mechanothermodynamics are presented and discussed.

2 Main provisions (I–XV) which constitute the foundation of the general theory of evolution of mechanothermodynamical systems are formulated.

2.1 The following theories and models are developed

• energy theory of limiting states,

• energy theory of damage,

• foundations of the theory of electrochemical damage,

• elements of the theory of translimiting states of mechanothermodynamical systems.

2.2 Hypothesis about the evolution of mechanothermodynamical systems (see Tables 1 and 2, Figures 7 and 8) was developed. This hypothesis describes the lifetime of the system as a whole (stage I) and the duration of its translimiting existence or degradation (stage II). It was established [see (151)] that the effective energy absorbed by the system in the process of reaching its limiting state is identically equal to the released (dissipated) effective energy in the process of its degradation up to the decomposition (for example, into atoms).

3 Methods and procedures for calculating effective (dangerous) energy which is spent on generation, motion and accumulation of irreversible damages are developed (see formulas (79)–(83) and corresponding text).

4 Fundamentals of the theory of Λ-interaction between damages caused by loads of different nature (mechanical, thermodynamic, etc.) (see formulas (69), (70), (161) and corresponding text) are given. This theory allows considering the influence of the spontaneous processes of hardening-softening on the state of damage of mechanothermodynamical systems.

5 Correlation of system damage with probability (Figure 5) in the course of its evolution is analyzed. Idea of reliable

probability $1 < P_* < \infty$ of damage at the stage of translimiting states is proposed.

6 Concept of ″arbitrarily″ (″infinitely″) large but of course finite number big (denoted by $\hbar$) [see formula (150)] is introduced. Its practical importance is shown for the analysis of the evolution of systems by damage [see formulas (152) and (153)].

7 Set of the physical characteristics and specification of the features of limiting states of objects and systems (see Tables 3 and 4) are given. They may be useful for the specialists in the relevant fields of research.

8 In practical terms, a single MTD function of critical by damage (limiting) states of metal and polymer materials operating in different conditions (formula (158) and Figure 20) is obtained in this work. The analysis of more than 600 experimental results (Figures 12, 13, 19, 21, 22) showed that this function is fundamental: it is valid for low-, average- and high-strength states of pure metals, alloys, and polymers over a wide range of temperatures of medium (from helium temperature to $0.8T_S$, where T_S is the material melting temperature) and mechanical stresses (up to the strength limit for single static loading) while the fatigue life was of the order of 10^6–10^8cycles. The fundamental MTD function as found in the present study can be used for effective prediction of the behavior of concrete MTD systems in different operating (test) conditions [procedures (162) and (163)]. Model (165), (166) is proposed for the description of the influence of thermal corrosion and corrosion at stress on the changes in the limiting states of materials.

Finally, it should be noted that the research in Mechanothermodynamics is just beginning. The deepening and broadening of the scope of works in this new and promising area of knowledge are expected in the near future. In the authors' opinion, the utility of works is so great that it is difficult to foresee their results.

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